Flow and oscillations in collapsible tubes: physiological applications and low-dimensional models.

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This lecture is an overview of its subject. The motivation comes from physiology: Air-flow in the lungs, where the phenomenon of flow limitation during forced expiration is a consequence of large-airway collapse, and wheezing is a manifestation of self-excited mechanical oscillations; Blood flow in veins, such as those of giraffes, in which the return of blood to the heart from the head must be accompanied by partial venous collapse, and in arteries, which exhibit self-excited oscillations (Korotkov sounds) when compressed by a blood-pressure cuff. Laboratory experiments are frequently conducted in a *Starling resistor*, a finite length of flexible tube, mounted between two rigid tubes and contained in a pressurised chamber. Steady conditions upstream and downstream give rise not only to steady flows, but also to a rich variety of self-excited oscillations [1], which theoreticians have been seeking to understand for at least four decades. Some of the observations have been reproduced in full Navier-Stokes computations for a two-dimensional model [2], but these do not provide physical understanding of the mechanisms that give rise to the oscillations.

In order to gain such understanding we seek a self-consistent mathematical model. We concentrate principally on 1D models, in which the key dependent variables are the cross-sectional area A and the cross-sectionally averaged velocity u and pressure p, all taken to be functions of longitudinal coordinate x and time t. The governing equations are those of conservation of mass and momentum and a tube law representing the elastic properties of the vessel. In the momentum equation, the viscous resistance term depends on the fluid velocity profile, which itself depends on both the geometry and time; this is conventionally modelled either as a linear function of fluid velocity, accurate at low Reynolds number, or with an ad hoc representation of the energy loss at flow separation. Even with such crude approximations, the 1D models agree quite well both with observations in the giraffe [3] and with some of the 2D computations and 3D experiments [4].

Nevertheless, it is desirable to make the model completely rational, at least for the 2D

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model problem, in which part of one wall of a parallel sided channel is replaced by a membrane under tension. One approach, for large Reynolds-number flow, and a long membrane, is to consider small deflections of the membrane and use interactive boundary-layer theory [5]. This leads to interesting predictions, such as the impossibility of simultaneously prescribing the flow rate and the upstream pressure, but not to oscillations, except in cases where wall inertia is important (flutter). Another approach is to assume a parabolic velocity profile everywhere, leading to a rational choice for the inertia and viscous terms in the 1D momentum equation [6]. If, further, the undisturbed membrane is taken to be flat, by a suitable choice of external pressure distribution, the system leads to an oscillatory instability even without wall inertia [7]. Whether these oscillations have the same physics as those computed numerically at lower Reynolds number remains to be seen.

References

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