Chapter 1

Dimensional Analysis and Correlations

In order to design unit operations in chemical engineering processes, it is necessary to obtain the relationship between the average transfer rates (mass, heat, momentum) and the applied forces that are used to drive the transport (concentration difference, temperature difference, velocity difference). Dimensional analysis is a very powerful tool that enables us to simplify this relationship and express it in a compact manner, through the use of dimensionless groups and correlations between these groups. A deeper understanding is obtained by interpreting the dimensionless groups as the ratio of fluxes and driving forces, or as the ratio of two driving forces. The magnitude of dimensionless groups enable us to differentiate between the dominant and the negligible forces in a transport process.

In this chapter, we first discuss dimensions and units, and obtain dimensionless groups of importance in transport processes using dimensional analysis. Dimensionless groups are then interpreted on the basis of either the flux-force ratio, or as the balance between two driving forces that they represent. The concept of mass, momentum and energy diffusion is then discussed, and the respective diffusivities are derived. Correlations between dimensionless groups for some important transport processes involving heat, mass and momentum transfer are then introduced. Some of these correlations, which are derived by more exact calculations in the reminder of the book, are introduced here. Some physical insight is provided into the nature of the correlations in different transport (diffusive, convective) and flow (laminar,turbulent) regimes.

1.1 Units

All physical quantities are expressed in two parts. The first is a unit, which indicates the basis for the measurement of the quantity, and the second is the number of units. A physical quantity cannot be completely specified without

indicating what the unit is. For example, one cannot say that the length of a box is 3 or the weight of an object is 2. It is also necessary to specify the units being adopted, that is feet or meters (in the case of length) and pounds or kilograms (in the case of weight). Though it is necessary to have only one unit for a quantity, for historical reasons there are often many different units adopted for the same quantity, for example foot and meters for length or pounds and kilograms for weight. These are often a hindrance, since it is necessary to convert from one application to another in practical applications. In our treatment of transport phenomena, we will restrict attention to the SI system of units (meters, kilograms, seconds) where it is necessary to specify units. To the extent possible, we will work with dimensionless groups, whose value is independent of the units used.

1.1.1 Fundamental and derived units

Units can also be divided into two categories — fundamental units and derived units. For example,

1. Length, which is expressed in feet or meters, can be considered a fundamental unit. Similarly, time, which is expressed in seconds or hours, is also a fundamental unit.

2. In contrast, velocity, which is expressed as the length moved per unit time, is a derived quantity, because its unit is the ratio of the units of length and time.

3. Similarly, density, which is the mass per unit volume, is a derived quantity, since it is expressed in terms of the ratio of mass and volume.

4. Volume is also a derived quantity, since it is expressed in terms of the length.

1.1.2 SI units

It turns out that all units for all physical quantities can be expressed in terms of a set of six fundamental units. By international convention, the standard system of units is the SI system, in which the fundamental measures and units are,

1. Mass($\mathcal{M}$) with unit kilogram.

2. Length($\mathcal{L}$) with unit meter.

3. Time($\mathcal{T}$) with unit second.

4. Temperature(Υ) with unit Kelvin.

5. Current with unit ampere.

6. Light intensity with unit candela.

Quantity	Dimension	Unit (SI)
Mass	$\mathcal{M}$	kilogram (kg)
Length	$\mathcal{L}$	meter (m)
Time	$\mathcal{T}$	second (s)
Temperature	Υ	K
Velocity	$\mathcal{L}\mathcal{T}^{-1}$	(m/s)
Acceleration	$\mathcal{L}\mathcal{T}^{-2}$	(m/s^2)
Force	$\mathcal{M}\mathcal{L}\mathcal{T}^{-2}$	(kg m/s^2)
Energy	$\mathcal{M}\mathcal{L}^2\mathcal{T}^{-2}$	(kg m^2/s^2) (Newton)
Work	$\mathcal{M}\mathcal{L}^2\mathcal{T}^{-2}$	(kg m^2/s^2) (Newton)
Heat	$\mathcal{M}\mathcal{L}^2\mathcal{T}^{-2}$	(kg m^2/s^2) (Newton)
Pressure	$\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-2}$	(kg/m/s^2) (Pascal)
Stress	$\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-2}$	(kg/m/s^2) (Pascal)
Elasticity	$\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-2}$	(kg/m/s^2) (Pascal)
Viscosity	$\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-1}$	(kg/m/s) (Poise)
Diffusivity	$\mathcal{L}^2\mathcal{T}^{-1}$	(m^2 s^{-1})
Thermal conductivity	$\mathcal{M}\mathcal{L}\mathcal{T}^{-3}\Upsilon^{-1}$	(kg m s^{-3} K^{-1}
conductivity		
Specific heat	$\mathcal{L}^2\mathcal{T}^{-2}\Upsilon^{-1}$	(m^2 s^{-2} K^{-1} (J/kg/K)
heat		

Table 1.1: Dimensions and units of some commonly used quantities in transport processes.

In the present course, we will only analyse quantities with the mass $\mathcal{M}$, length $\mathcal{L}$, time $\mathcal{T}$ and temperature Υ dimensions. All other physical quantities can be expressed in terms of these fundamental quantities. Some of the important quantities encountered in chemical engineering applications and their units in the SI systems are given in the following table 1.1.

The derived units of all other quantities are determined from relations that define these quantities. For example

1. The unit of **Force** is determined from Newton's third law, which states that force on an object is mass times acceleration $F = ma$. Therefore, the unit of force (Newton) is the unit of mass times $\mathcal{M}$ (kg) times the unit of acceleration $\mathcal{L}\mathcal{T}^{-2}$ (m/s^2).

2. **Pressure** is the force acting per unit area of the surface. The unit of force is $\mathcal{M}\mathcal{L}\mathcal{T}^{-2}$, and the unit of area is $\mathcal{L}^2$, and so the unit of pressure is $\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-2}$.

3. The **density** is the mass of a unit volume of the material, so the density has dimensions of mass per unit volume, $\mathcal{M}\mathcal{L}^{-3}$.

4. **Concentration** is the mass of solute per unit volume of solution, so it also has dimensions of mass per unit volume, $\mathcal{M}\mathcal{L}^{-3}$.

5. The **mass flux** j is the mass transported across a surface per unit area per unit time, with dimensions of $\mathcal{M}\mathcal{L}^{-2}\mathcal{T}^{-1}$.

6. The **diffusion coefficient** is the proportionality constant in Fick's law for diffusion. If we take a solution of thickness L, and the difference in the concentration on the two ends is Δc, then the mass flux is given by,

$$j = D\frac{\Delta c}{L} \tag{1.1}$$

where Δc is the concentration difference across a distance Δx. The mass flux has units of $\mathcal{M}\mathcal{L}^{-2}\mathcal{T}^{-1}$, and the the concentration gradient has dimensions of $\mathcal{M}\mathcal{L}^{-4}$, and therefore, the diffusion coefficient has dimensions of $\mathcal{L}^2\mathcal{T}^{-1}$.

7. The heat flux q is the energy transferred per unit area per unit time. The unit of energy is $\mathcal{M}\mathcal{L}^2\mathcal{T}^{-2}$, and so the unit of energy flux is $\mathcal{M}\mathcal{T}^{-3}$.

8. **Specific heat:** The change in thermal energy ΔE is related to the change in temperature of an object ΔT as follows

$$\Delta E = mC\Delta T \tag{1.2}$$

In the above equation, ΔE has dimensions of energy $\mathcal{M}\mathcal{L}^2\mathcal{T}^{-2}$, mass has dimension $\mathcal{M}$ and temperature has dimension Υ. Therefore, the specific heat has dimension $\mathcal{L}^2\mathcal{T}^{-2}\Upsilon^{-1}$, or units ($\mathrm{m}^2/\mathrm{s}^2/\mathrm{K}$).

9. The **thermal conductivity** is obtained from Fouriers law for heat conduction. If we consider a slab of material of thickness L subjected to a temperature difference ΔT, the heat flux is given by,

$$q = -k\frac{\Delta T}{L} \tag{1.3}$$

and has dimensions of $\mathcal{M}\mathcal{L}\mathcal{T}^{-3}\Upsilon^{-1}$.

10. The **stress** has dimensions of force per unit area, $\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-2}$.

11. **Viscosity:** The unit of viscosity can be determined from Newton's law for viscosity for a fluid between two plates separated by a distance L having a relative velocity V as in figure 1.1.2.

$$\tau = \mu V/L \tag{1.4}$$

In the above equation, τ is the shear stress on the plates, and has dimensions of force per unit area $\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-2}$, the velocity has dimensions of $\mathcal{L}\mathcal{T}^{-1}$ and distance between the plates has dimensions of $\mathcal{L}$. Therefore, the viscosity has dimensions of $\tau L/V$, which is $\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-1}$.

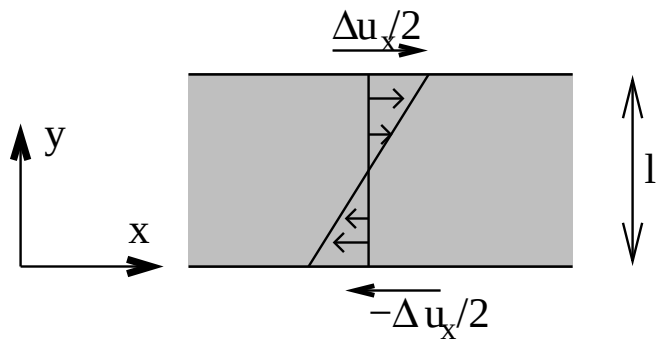


Figure 1.1: The fluid flow between two plates separated by a distance L moving with velocity $(\pm V/2)$ in the tangential direction.

1.2 Dimensional analysis

The first step in dimensional analysis is to identify the transport properties (fluid velocity, heat flux, mass flux, etc.) and the material properties (viscosity, thermal conductivity, specific heat, density, etc.) which are relevant to the problem, and list out all these quantities along with their dimensions. In general, if a problem contains n dynamical and material parameters, and these contain m fundamental dimensions, then there are $n - m$ independent dimensionless groups. This important principle is called the *Buckingham Pi Theorem.*

The choice of the independent dimensionless groups is subjective. For example, if Φ is a dimensionless group, then $\Phi^2, \Phi^{-3/2}$, etc. are also dimensionless group. Therefore, in assembling a dimensionless group, the power of one of the important variables can be decided arbitrarily, and all other dependencies are determined in terms of this one. In addition, if Φ and Ψ are two dimensionless groups, then the product $\Phi \times \Psi$ is also a dimensionless group. Though there is some arbitrariness in assembling the dimensionless groups, the number of such independent dimensionless groups is still $n - m$, if n is the total number of dimensional quantities in the problem and m is the number of dimensions.

1.2.1 Sphere falling in a fluid

Consider the example of a sphere falling in a tank containing the fluid1.2.1. The equation of motion for the sphere velocity in the vertical direction can be written as,

$$m\frac{dU}{dt} = mg - F_D \tag{1.5}$$

where m and U are the mass and velocity of the sphere, g is the acceleration due to gravity and F_D is the drag force. When the sphere attains its terminal velocity, the drag force is exactly balanced by the gravitational force. Our task

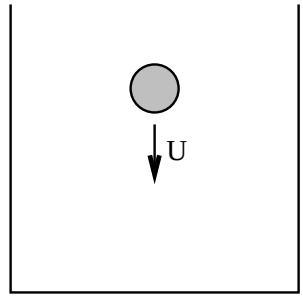


Figure 1.2: A sphere falling in a tank containing a liquid.

is to obtain an expression for the drag force as a function of the sphere velocity and the fluid properties.

The motion of the sphere disturbs the fluid around it, and causes fluid flow. This fluid flow results in friction, and this frictional drag exerts a force on the sphere. Since the frictional resistance to flow is due to viscous stresses, it is expected that the viscosity μ will be important in determining the frictional force. In addition, the force is expected to depend on the velocity of the sphere U, since a higher speed would result in a larger force. The length scales that could affect the flow are the width of the tank L and the radius of the sphere R. The density of the fluid ρ could also be important, since a higher force is required for accelerating a fluid with a higher density.

It is important to note that the mass of the sphere or its density should not be relevant determining the flow, once the sphere velocity is specified. This is because the fluid velocity around the sphere, which results in the frictional force, is determined by the velocity with which the sphere is moving, and the mass or density of the sphere does not affect the fluid velocity. Similarly, the acceleration due to gravity determines the gravitational force on the sphere, but does not directly affect the fluid velocity around the sphere, once the sphere velocity is specified. Therefore, it is necessary to apply some judgement at the start of dimensional analysis to distinguish the material and dynamical properties that influence in the desired quantity. It is essential to ensure that no irrelevant quantity is included in the analysis.

Now that all the relevant dimensional quantities for the drag force have been determined, it is necessary to examine which are the *important* ones. This is a skill, and can be developed only by practice. In this particular case, let us look at the length scales that are likely to be important. If the width and height of the container are large compared to the radius of the sphere, it is likely that only the radius of the sphere is a relevant length scale. On the other hand, if the sphere is very close to one of the walls of the container, say within a few radii, then the distance from the wall as well as the radius of the sphere could be important. For definiteness, let us proceed with the assumption that only the radius of the sphere is important in this case. This results in a set of important

variables

1. Radius of sphere R
2. Velocity of sphere U
3. Viscosity of fluid μ
4. Density of fluid ρ

So an equation for the force has to have the form

$$F = F(\rho, R, U, \mu) \tag{1.6}$$

The dimensions of the quantities on the two sides are

$$F \rightarrow \mathcal{M}\mathcal{L}\mathcal{T}^{-2} \tag{1.7}$$

$$\rho \rightarrow \mathcal{M}\mathcal{L}^{-3} \tag{1.8}$$

$$R \rightarrow \mathcal{L} \tag{1.9}$$

$$U \rightarrow \mathcal{L}\mathcal{T}^{-1} \tag{1.10}$$

$$\mu \rightarrow \mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-1} \tag{1.11}$$

Since there are five dimensional quantities and three dimensions involved in the above relationship, there are two dimensionless groups. One of these groups has to involve the force, while the other is a combination of the sphere diameter, velocity, and fluid properties (density and viscosity).

Let us construct the first dimensionless group, which we call a scaled force Φ_{FV}, as a combination of the force, viscosity, velocity and

$$\Phi_{FV} = F\mu^a U^b R^c \tag{1.12}$$

where a, b and c are the desired indices which render Φ_{FV} dimensionless. The dimensions of the left and right sides of the equation are

$$\mathcal{M}^0\mathcal{L}^0\mathcal{T}^0 = (\mathcal{M}\mathcal{L}\mathcal{T}^{-2})(\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-1})^a(\mathcal{L}\mathcal{T}^{-1})^b\mathcal{L}^c \tag{1.13}$$

For the dimensions on two sides to be equal, we require that

$$\mathcal{M} : a + 1 = 0 \tag{1.14}$$

$$\mathcal{L} : -a + b + c + 1 = 0 \tag{1.15}$$

$$\mathcal{T} : -a - b - 2 = 0 \tag{1.16}$$

These equations can be solved simultaneously, to obtain $a = -1, b = -1, c = -1$. Thus, the first dimensionless group is,

$$\Phi_F = \frac{F}{\mu R U} \tag{1.17}$$

The second dimensionless group, which we call Re, is a combination of the density, diameter, sphere velocity and viscosity. We write this as,

$$\mathrm{Re} = \rho U^a R^b \mu^c \tag{1.18}$$

In defining the dimensionless group, the exponent of any one of the quantities can be set equal to 1 without loss of generality. In equation **??**, we have chosen to set the exponent of the density equal to 1, to bring it in accordance with the conventional definition of the Reynolds number defined a little later. The dimensions of the quantities on the right side of equation **??** are,

$$(\mathcal{M}\mathcal{L}^{-3})(\mathcal{L}\mathcal{T}^{-1})^a \mathcal{L}^b (\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-1})^c \tag{1.19}$$

For dimensional consistency, we have three relations

$$\mathcal{M} : 1 + c = 0 \tag{1.20}$$

$$\mathcal{L} : -3 + a + b - c = 0 \tag{1.21}$$

$$\mathcal{T} : -a - c = 0 \tag{1.22}$$

These equations can be solved to obtain $a = 1$, $b = 1$ and $c = -1$. Therefore, the dimensionless number Ψ is

$$\mathrm{Re} = (\rho U R/\mu) \tag{1.23}$$

The relation between force and velocity can also be written in this case as

$$\Phi_{FV} = F/(\mu R U) = G(\mathrm{Re}) \tag{1.24}$$

where G is a function of the dimensionless group $\mathrm{Re} = (\rho U R/\mu)$.

In equation **??**, the force has been non-dimensionalised by the 'viscous force scale' $(\mu R U)$. Instead of using the viscosity, radius and velocity do non-dimensionalise the force, we could have chosen to use the density, radius and velocity instead. This would have lead to the dimensionless group $\Phi_{FI} = (F/(\rho U^2 R^2))$, where $(\rho U^2 R^2)$, the 'inertial force scale', is a suitable combination of density, velocity and radius with dimensions of force. It is easily verified that the dimensionless forces scaled in these two different ways are related by,

$$\Phi_{FV} = \mathrm{Re}\Phi_{FI} \tag{1.25}$$

The Reynolds number is then a ratio of the inertial and viscous force scales, $\mathrm{Re} = (\rho U^2 R^2)/(\mu R U)$. That is why the Reynolds number is often referred to as the balance between the viscous and inertial forces.

Further simplifications can be made in some limiting cases. When the group $(\rho U R/\mu)$ is very small, the fluid flow is dominated by viscous effects, the drag force should not determine on the inertial force scale. Therefore, the drag force has the form $\Phi_{FV} = \mathrm{Constant}$, or

$$F_D = \mathrm{Constant}\mu U R \tag{1.26}$$

Quantity	Symbol	Dimension	Modified dimension
Mass flux (mass/area/time)	j	$\mathcal{M}\mathcal{L}^{-2}\mathcal{T}^{-1}$	$\mathcal{M}_s\mathcal{L}^{-2}\mathcal{T}^{-1}$
Diffusion coefficient	D	$\mathcal{L}^2\mathcal{T}^{-1}$	
Concentration difference	Δc	$\mathcal{M}\mathcal{L}^{-3}$	$\mathcal{M}_s\mathcal{L}^3$
Particle diameter	D	$\mathcal{L}$	
Particle velocity	U	$\mathcal{L}\mathcal{T}^{-1}$	
Density of fluid	ρ	$\mathcal{M}\mathcal{L}^{-3}$	
Viscosity of fluid	μ	$\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-1}$	

Table 1.2: Relevant quantities and their dimensions for the mass transfer to a particle.

The limitation of dimensional analysis is that the value of the constant cannot be determined, and more detailed calculations in chapter **??** reveal that the exact value of the constant is 6π. This leads to Stokes drag law for the force on a sphere in the limit of small Reynolds number, $F_D = 6\pi\mu RU$.

The Reynolds number can be physically interpreted as the ratio of inertial and viscous forces, or as the ratio of convection and diffusion. The latter interpretation is more useful, since it has analogies in mass and heat transfer processes. As we shall see in detail in the next chapter, the diffusion coefficient for momentum, is the 'kinematic viscosity', $\nu = (\mu/\rho)$, where ρ is the mass density. It is easy to verify that the kinematic viscosity has the same dimensions as the mass diffusion coefficient, $\mathcal{L}^2\mathcal{T}^{-1}$. The Reynolds number can be written as $\mathrm{Re} = (UR/\nu)$, which is the ratio of convective transport of momentum (due to the fluid velocity) and diffusive transport due to momentum diffusivity.

This type of analysis could be extended to more complicated problems. For example, if the particle falling is not a sphere but a more complicated object, such as a spheroid, then there are two length scales, the major and minor axes of the spheroid, R_1 and R_2. In this case, the number of variables increases to 6, and there are three dimensional groups. One of these could be assumed to be the ratio of the lengths (aspect ratio) (R_1/R_2).

1.2.2 Mass transfer to a suspended particle:

Consider a stirred tank reactor for heterogeneous catalysis, shown in figure **??**, where the reactants and products are in solution and the catalyst is in the form of solid particles. The dissolved reactant R diffuses to the surface of the suspended catalyst particle, reacts at the surface and the product diffuses back into the fluid. It is necessary to determine the average flux of the reactant to the surface, given the difference in concentration $\Delta c = (c_\infty - c_s)$, where c_∞ is the concentration in the bulk and c_s is the surface concentration. There is also relative motion of characteristic velocity U between the catalyst particle and the fluid, due to stirring.

The different dimensional quantities of relevance are, The choice of dimensional quantities requires further discussion. It is clear that the average mass

flux depends on the bulk concentration and the diffusion coefficient, and the particle diameter. However, it is also necessary to include the fluid density, viscosity and the particle velocity relative to the fluid for the following reason. When the particle moves relative to the fluid, the flow pattern generated alters the distribution of the solute around the particles, and thereby modifies the diffusion flux at the particle surface. The flow pattern, in turn, depends on the fluid viscosity and density and the flow velocity, and therefore, the average mass flux could also depend on these.

In table 1.3, there are seven dimensional quantities and three dimensions. On this basis, we would expect that there are four dimensionless groups. However, a further simplification can be made by distinguishing between the mass dimension in the mass transport (which involves the solute mass) and that in the flow dynamics (which involves the mass of the total fluid). The flux and the diffusion coefficient contain the mass of the solute, whereas the mass of the fluid (solute plus solvent) appears in the density and fluid viscosity. If the solute concentration does not affect the fluid density and viscosity, we can make a distinction between the mass dimension for the solute, $\mathcal{M}_s$, from the mass dimension for the fluid, $\mathcal{M}$. In the modified dimensions shown in table 1.3, the mass flux and the diffusion coefficient depend on the solute mass $\mathcal{M}_s$, while the density and viscosity depend on the fluid mass $\mathcal{M}$. There are now four dimensions, $\mathcal{M}$, $\mathcal{M}_s$, $\mathcal{L}$ and $\mathcal{T}$, and therefore there are only three dimensionless groups.

The first dimensionless group can be constructed by non-dimensionalising the flux by the diffusion coefficient, the concentration difference and the particle diameter,

$$\Phi_j = j(\Delta c)^a \mathcal{D}^b D^c \tag{1.27}$$

The indices a, b and c are determined from dimensional consistency to provide the dimensionless flux, called the Sherwood number,

$$\mathrm{Sh} = \frac{jD}{\mathcal{D}\Delta c} \tag{1.28}$$

Two other dimensionless groups can be defined. One of the dimensionless numbers if the Reynolds number, $\mathrm{Re} = (\rho U D/\mu)$, which is the ratio of fluid inertia and viscosity. The second dimensionless group can be defined in two ways. One possible definition is the Schmidt number, $\mathrm{Sc} = (\mu/\rho\mathcal{D})$, which is the dimensionless combination of the diffusivity, viscosity and density. The alternate dimensionless group is the Peclet number, $\mathrm{Pe} = (UD/\mathcal{D})$, constructed from the flow velocity, diameter and the mass diffusivity. If we use the Reynolds and Schmidt numbers, the dimensionless flux can be expressed as,

$$\Phi_j = \mathrm{Function}(\mathrm{Re}, \mathrm{Sc}) \tag{1.29}$$

1.3 Heat transfer in a pipe

This is a slightly more complicated problem. A fluid is flowing through a pipe, which is heated from outside. The temperature of the wall of the pipe is higher

Quantity	Symbol	Dimension
Heat flux	q	$\mathcal{H}\mathcal{T}^{-1}\mathcal{L}^{-2}$
Diameter of pipe	D	$\mathcal{L}$
Length of pipe	l	$\mathcal{L}$
Average fluid velocity	U	$\mathcal{L}\mathcal{T}^{-1}$
Density of fluid	ρ	$\mathcal{M}\mathcal{L}^{-3}$
Viscosity of fluid	μ	$\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-1}$
Specific heat of fluid	c_p	$\mathcal{H}\mathcal{M}^{-1}\Upsilon^{-1}$
Thermal conductivity	k	$\mathcal{H}\mathcal{L}^{-1}\mathcal{T}^{-1}\Upsilon^{-1}$
Temperature difference	ΔT	Υ

Table 1.3: Relevant quantities and their dimensions for the heat transfer to a fluid flowing in a pipe.

than the average temperature of the fluid by a constant value ΔT. The change in temperature of the fluid is due to the heat transfer from the walls, and not due to frictional heating generated by the flow. One would like to predict the rate of transfer of heat, per unit area of the wall of the pipe, so that the length of pipe required for the heat exchanger can be designed.

The first step is to collect the set of variables on which the heat flux q can depend. This can depend on the thermal properties of the fluid, the conductivity k and the specific heat c_p, the difference in temperature between the fluid and the wall ΔT, the flow properties of the fluid, the density ρ and viscosity μ, and the average fluid velocity U, and the diameter of the pipe D, and length of pipe L. The fundamental dimensionless groups that we can classify these into are $\mathcal{M}$, $\mathcal{L}$, $\mathcal{T}$ and temperature Υ.

There are nine dimensionless groups and four dimensions in table 1.3, apparently necesitating five dimensionless groups. However, a further reduction is possible when there is no interconversion between heat energy (which is being transferred) and mechanical energy (which is driving the flow). In this case, it is possible to consider heat energy as a dimension $\mathcal{H}$ which is different from mechanical energy. There are now five dimensions, $\mathcal{H}$, $\mathcal{M}$, $\mathcal{L}$, $\mathcal{T}$ and Υ, and a total of nine dimensional quantities in table 1.3. Therefore, there are only four dimensionless groups.

Of the dimensionless groups, the easiest one to is the ratio (L/D) of the length and diameter of the pipe. The dimensionless group containing q is the 'dependent' dimensionless group, which contains the dependent variable which has to be determined as a function of all the other 'independent' variables in the problem. For the independent variable q, the relation is of the form

$$qD^a k^b \Delta T^c \mu^d \rho^e = \text{Dimensionless} \tag{1.30}$$

In dimensional form, the equation for these become

$$\mathcal{H}\mathcal{L}^{-2}\mathcal{T}^{-1}(\mathcal{L})^a(\mathcal{H}\mathcal{L}^{-1}\mathcal{T}^{-1}\Upsilon^{-1})^b(\Upsilon)^c(\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-1})^d(\mathcal{M}\mathcal{L}^{-3})^e = \mathcal{H}^0\mathcal{M}^0\mathcal{L}^0\mathcal{T}^0\Upsilon^0 \tag{1.31}$$

The five equations required for the consistency of the above dimensional relationship are

$$\mathcal{H} : b = -1 \tag{1.32}$$

$$\mathcal{M} : d + e = 0 \tag{1.33}$$

$$\mathcal{L} : a - b - d - 3e = 2 \tag{1.34}$$

$$\mathcal{T} : -b - d = 1 \tag{1.35}$$

$$\Upsilon : -b + c = 0 \tag{1.36}$$

The above relations can easily be solved to obtain $a = 1$, $b = -1$, $c = -1$, $d = 0$ and $e = 0$. Using these, the dimensionless group is

$$\mathrm{Nu} = \frac{qD}{k\Delta T} \tag{1.37}$$

Of the two other dimensionless groups, one can easily be identified as the Reynolds number, $\mathrm{Re} = (\rho U D/\mu)$, which is the ratio of inertia and viscosity for this case. The second dimensionless group contains the specific heat c_p, which has not been employed so far. Since c_p contains both the thermal energy and mass dimensions, the dimensionless group has to contain the thermal conductivity k, as well as the (viscosity μ or density ρ). The dimensionless group constructed with the specific heat, viscosity and conductivity is the Prandtl number,

$$\mathrm{Pr} = \frac{c_p \mu}{k} \tag{1.38}$$

Therefore, the general expression for the average heat flux can be written as,

$$\frac{qD}{k\Delta T} = \Phi\left(\frac{L}{D}, \frac{\rho U D}{\mu}, \frac{c_p \mu}{k}\right) \tag{1.39}$$

Dimensional analysis has certainly simplified the problem, since it is much easier to deal with relationships between four dimensionless groups rather than with nine dimensional variables. However, it is not possible to obtain further simplification using dimensional analysis. There are two possible ways to further simplify the problem. One is to do further analytical calculations which incorporate the details of the heat and mass transfer processes. The other is to do experiments and obtain empirical correlations between the parameters. In the latter case, it is sufficient to consider the variation in the heat flux for variations in the dimensionless groups alone, and it is not necessary to examine variations in individual parameters.

1.4 Dimensionless groups

Dimensionless groups can be classified into three broad categories, the dimensionless fluxes, the ratios of convection and diffusion, and the ratios of different types of diffusion. Before proceeding to define dimensionless groups, we first

define the diffusion coefficients for mass, momentum and energy, in order to provide a definite basis for the discussion. The molecular origins of diffusion will be discussed in the next chapter; here, we restrict ourselves to the macroscopic definition.

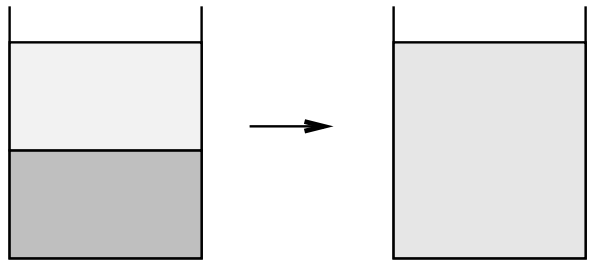
1.4.1 Mass, momentum and energy diffusivities:

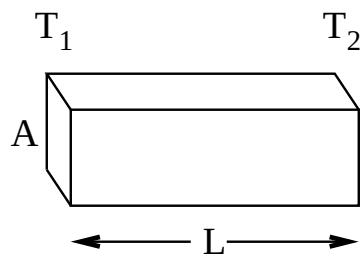
The fundamental constitutive relations in transport processes are the Fick's law for mass diffusion, Fourier's law for heat conduction and Newton's law for viscosity. The respective diffusion coefficients can be defined as follows.

1. When there is a concentration difference maintained across a slab of fluid, as shown in figure 1.4.1, there is transfer of mass from the surface at higher concentration to the surface at lower concentration. As shown in figure 1.4.1 the mass flux (mass per unit area per unit time) is inversely proportional to the length and directly proportional to the difference in concentration or temperature across the material. If a concentration difference Δc is maintained between two ends of a slab of length l, the mass flux j per unit area, which has units of (mass / area / time), is given by Fick's law,
$$j_c = -\mathcal{D}\frac{\Delta c}{l} \tag{1.40}$$
Here, the negative sign indicates that mass is transferred from the region of higher concentration to the region of lower concentration.

2. For the transport of heat, the heat flux is related to the temperature difference by Fick's law. For a slab of material of length l with a temperature difference ΔT across the material, (figure **??**), the heat flux q is given by Fourier's law,
$$q = -k\frac{\Delta T}{l} \tag{1.41}$$
where k is the thermal conductivity, and the negative sign indicates that heat is transferred from the region of higher to the region of lower temperature.

3. The Newton's law of viscosity relates the shear stress τ (force per unit area at the wall) to the strain rate (change in velocity per unit length across the flow) for the simple shear flow of a fluid as shown in figure 1.1.2.
$$\tau_{xy} = \mu\frac{\Delta u_x}{l} \tag{1.42}$$
It should be noted that there is no negative sign in the above Newton's law 1.42, in contrast to Fick's law and Fourier's law. This is due to the difference in convention with regard to the definition of stress in fluid mechanics, and the definition of fluxes in transport phenomena. The shear stress τ_{xy} in Newton's law is defined as the force per unit area at a surface





in the x direction whose outward unit normal is in the y direction. In contrast, the fluxes are defined as positive if they are directed into the volume. Therefore, the shear stress is actually the negative of the momentum flux. If the stress is defined to the the force per unit area acting at a surface whose inward unit normal is in the x direction; this would introduce a negative sign in equation 1.42. However, it is conventional in fluid mechanics to define the stress with reference to the outward unit normal to the surface. As we will see later in this course, this difference in convention will not affect the balance equations that are finally obtained for the rate of change of momentum.

In this course, we will adopt the convention of defining the stress τ_{xy} as the force per unit area in the x direction acting along a surface whose outward unit normal is in the y direction, and use equation 1.42 for Newton's law of viscosity.

The diffusion coefficients are the proportionality constants in the relationship between the flux of a quantity (mass, heat, momentum) and the driving force. The **flux** of a quantity (mass, heat, momentum) is the amount of that quantity transferred per unit area per unit time. The *driving force* for a quantity (mass, heat, momentum) is the gradient (change per unit distance) in the density (quantity per unit volume) of that quantity. So the transport equations can be

written in the general form,

$$\begin{pmatrix} \text{Transport of } \textbf{quantity} \\ \text{per unit area} \\ \text{per unit time} \end{pmatrix} = \begin{pmatrix} \text{Diffusion} \\ \text{coefficient} \end{pmatrix} \times \frac{\begin{pmatrix} \text{Change in density (per unit volume)} \\ \text{of the } \textbf{quantity} \\ \textbf{acrossthematerial} \end{pmatrix}}{\text{Thickness of material}} \tag{1.43}$$

From dimensional analysis of the above equation, it is easy to see that the diffusion coefficients of all quantities have dimensiions of $\mathcal{L}^2\mathcal{T}^{-1}$ These diffusion coefficients are defined from Fick's law, Fourier's law and Newton's law as follows.

1. From Fick's law, the diffusion coefficient $\mathcal{D}$ is the ratio of mass flux (mass transported per unit area per unit time) and the gradient in the concentration (mass per unit volume). Therefore, the diffusivity of mass is just the diffusion coefficient $\mathcal{D}$.

2. It is possible to define a diffusion coefficient for heat transfer as follows. The difference in temperature ΔT can be expressed in terms of the difference in the specific energy between the two sides as $\Delta T = (\Delta E/\rho c_v)$ where ΔE is the specific energy (per unit volume). With this, the equation for the heat flux can be written as

$$j_e = \frac{k}{\rho c_v}\frac{\Delta E}{l} \tag{1.44}$$

   It is obvious that the above equation has the same form as the mass flux equation, with a **thermal diffusivity** $\mathcal{D}_H = (k/\rho c_p)$ which has units of $\mathcal{L}^2\mathcal{T}^{-1}$.

3. The 'momentum diffusivity' is the relation between the flux of momentum (rate of transport of momentum per unit area per unit time) and the difference in the momentum density (momentum per unit volume). Consider the layer of fluid shown in figure 1.1.2. Since the momentum of a parcel of fluid is the mass of that parcel multiplied by its velocity, the momentum density is the product of the mass density ρ and velocity u_x. Therefore, the equation for the flux 1.43, expressed in terms of the momentum density, is

$$\tau_{xy} = \nu\frac{\Delta(\rho u_x)}{l} \tag{1.45}$$

   where ν is the momentum diffusivity. It is clear, by comparing 1.42 and 1.45, that for an incompressible fluid with constant density (which we shall be concerned with in this course), the **momentum diffusivity** $\nu = (\mu/\rho)$. The momentum diffusivity ν, which has dimensions $\mathcal{L}^2\mathcal{T}^{-1}$, is also referred to as the 'kinematic viscosity'.

The dimensionless numbers which are ratios of diffusivities are

	Expression	Ratio
Reynolds number	$\frac{\rho U L}{\mu}$	$\frac{\text{Momentum convection}}{\text{Momentum diffusion}}$
Prandtl number	$\frac{\nu}{\mathcal{D}_H}$	$\frac{\text{Momentum diffusion}}{\text{Thermal diffusion}}$
Schmidt number	$\frac{\nu}{\mathcal{D}}$	$\frac{\text{Momentum diffusion}}{\text{Mass diffusion}}$
Peclet number	$\frac{Ul}{\mathcal{D}_H}$	$\frac{\text{Heat convection}}{\text{Heat diffusion}}$

Table 1.4: Dimensionless numbers as ratio of forces.

1. the Prandtl number $\mathrm{Pr} = (\nu/\mathcal{D}_H) = (c_p\mu/k)$, the ratio of momentum and thermal diffusivity, and

2. the Schmidt number $\mathrm{Sc} = (\mu/\rho\mathcal{D}) = (\nu/\mathcal{D})$, the ratio of the momentum and mass diffusivity.

1.4.2 Ratio of convection and diffusion:

Convective transport takes place due to the mean flow of a fluid, even in the absence of a concentration difference. For example, if a fluid with concentration c travels with velocity U in a pipe, the total amount of mass transported per unit time is (cUA_p), where A_p is the cross-sectional area of the tube. Therefore, the flux (mass transported by the fluid, per unit area perpendicular to the flow per unit time), is (cU). Consequently, the ratio between the rate of transport due to convective and diffusive effects is $(Ul/\text{Diffusivity})$, where l is the length scale across which there is a change in the density of this quantity.

The dimensionless numbers which are ratios of convective and diffusive transport rates (table 1.4.2) are,

1. the Reynolds number, $\mathrm{Re} = (\rho Ul/\mu) = (Ul/\nu)$, the ratio of momentum convection and diffusion,

2. the Peclet number for mass transfer $\mathrm{Pe} = (Ul/\mathcal{D})$, the ratio of mass convection and diffusion, and

3. the Peclet number for heat transfer, $\mathrm{Pe} = (Ul/\mathcal{D}_H) = (\rho C_p Ul/k)$.

1.4.3 Dimensionless numbers in natural convection

In natural convection, the driving force for convection is the body force caused by a variation in the density of the fluid, which is in turn caused by variation in temperature. In contrast to natural convection, the characteristic flow velocity is not known a priori, but has to be determined from the driving force due to the density difference. If we assume there is a balance between the driving force and the viscous stresses in the fluid, dimensional analysis can be used to infer that the characteristic velocity should scale as $U_c \propto (fD^2/\mu)$, where f is the driving force for convection per unit volume of fluid, D is the diameter of the body or the characteristic length, and μ is the viscosity.

The driving force for convection, per unit volume of the fluid, is proportional to the product of the density variation $\Delta\rho$ and the acceleration due to gravity g. The density variation is is proportional to $\rho\beta\Delta T$, where ΔT is the variation in temperature and β is the coefficient of thermal expansion. Therefore, the force per unit area is proportional to $\rho g\beta\Delta T$. With this, the characteristic velocity for the flow is $U_c \propto (\rho D^2 g\beta\delta T/\mu)$. The 'Grashof number' is the Reynolds number based upon this convection velocity,

$$\mathrm{Gr} = \frac{\rho U_c D}{\mu} = \frac{\rho^2 D^3 g\beta\Delta T}{\mu^2} \tag{1.46}$$

In defining the Grashof number, we used the momentum diffusivity and the body force to obtain the characteristic velocity scale. An alternative definition is the Rayleigh number, where the thermal diffusivity $(k/\rho C_p)$ is used to determine the characteristic velocity. The characteristic velocity is then given by $U_c \propto (fD^2\rho C_p/k)$, where $f = \rho\beta\Delta T$ is the driving force for convection per unit volume of fluid. The Reynolds number based on this convection velocity and momentum diffusivity is the Rayleigh number,

$$\mathrm{Ra} = \frac{\rho^2 C_p D^3 g\beta\Delta T}{\mu k} \tag{1.47}$$

The 'Rayleigh-Benard' instability of a fluid layer heated from below occurs when the Rayleigh number increases beyond a critical value.

1.4.4 Other dimensionless groups:

There are two important dimensionless groups involving surface tension, which are the Weber number and the Capillary number. The Weber number is the ratio of inertial and surface tension forces,

$$\mathrm{We} = \frac{\rho U^2 L}{\gamma} \tag{1.48}$$

while the capillary number is the ratio of viscous and surface tension forces,

$$\mathrm{Ca} = \frac{\mu U}{\gamma} \tag{1.49}$$

In problems involving free interfaces, if the Reynolds number is low, inertial forces are negligible compared to viscous forces, and the Capillary number has to be determined in order to assess the ratio of viscous and surface tension forces. At high Reynolds number, it is appropriate to determine the Weber number in order to examine the ratio of inertial and surface tension forces. In addition to the Weber and Capillary numbers, the Bond number is a dimensionless group appropriate for situations where a free interface is under the influence of a gravitational field. This dimensionless group is defined as the ratio of gravitational and surface tension forces,

$$\mathrm{Bo} = \frac{\rho g L^2}{\gamma} \tag{1.50}$$

1.4.5 Dimensionless groups involving gravity

The ratio of inertial and gravitational forces is given by the Froud number,

$$\mathrm{Fr} = \frac{U^2}{gL} \tag{1.51}$$

where U is the velocity, and g is the acceleration due to gravity, and L is the characteristic length. In applications involving rotation of fluids, the Froud number is also the ratio of centrifugal and gravitational forces.

1.5 Mass and heat transfer correlations:

1.5.1 Dimensionless fluxes:

In mass and heat transfer, non-dimensional fluxes are obtained by scaling the average flux by ((Diffusion coefficient) × (concentration/ temperature difference) / (Characteristic length)). The dimensionless group for mass transfer is the 'Sherwood number',

$$\mathrm{Sh} = \frac{j}{(\mathcal{D}\delta c/L)} \tag{1.52}$$

The above dimensionless number was derived using dimensional analysis for the mass transfer to a catalyst particle in section **??**. The ratio of the flux and the concentration difference, $j/\Delta c$, is called the 'mass transfer coefficient' k_m. Therefore, the Sherwood number is also written as

$$\mathrm{Sh} = \frac{k_m L}{\mathcal{D}} \tag{1.53}$$

Analogous to the Sherwood number, the Nusselt number is a dimensionless heat flux,

$$\mathrm{Nu} = \frac{q}{(\mathcal{D}_H \Delta e)/L} \tag{1.54}$$

where $\mathcal{D}_H$ is the thermal diffusivity, and Δe is the difference in the energy density for a characteristic length l. Since $\mathcal{D}_H = (k/\rho C_p)$ and $e = \rho C_p T$, the Nusselt number becomes,

$$\mathrm{Nu} = \frac{q}{(k\Delta T/L)} \tag{1.55}$$

This is what we had derived earlier using dimensionless analysis for heat transfer in a heat exchanger in section **??**. The ratio $(q/\Delta T)$ is also referred to as the 'heat transfer coefficinet' h, and the Nusselt number is written as

$$\mathrm{Nu} = (hL/k) \tag{1.56}$$

It is important to note that heat and mass transfer coefficients, h and k_m, are not material properties. They do depend on the system geometry and flow conditions, and can be calculated only after solving the specific transport problem. Therefore, they are derived quantities, on par with the Nusselt and Sherwood numbers. The calculation or experimentation required to obtain these is identical to that for the Nusselt and Sherwood numbers. Therefore, in this course, we shall work in terms of the Nusselt and Sherwood numbers alone, and will avoid the confusion generated by introducing the heat and mass transfer coefficients. In case a the heat and mass transfer coefficients are necessary, they can be calculated from the Nusselt and Sherwood numbers using equations **??** and **??**.

The length and velocity in dimensionless numbers for commonly encountered configurations are defined as follows.

1. For the flow past suspended particles, the velocity U is the difference between the particle velocity and the free-stream fluid velocity, and the length is usually the particle diameter D (for spehrical particles) or characteristic dimension.

2. For the flow past a flat plate, the length is the total length of the plate, while the velocity is the constant free-stream velocity far from the plate. The average fluxes are defined per unit surface area of the plate.

3. For the flow in tubes and channels, the length is the tube diameter or channel width, while the velocity is the average flow velocity through the tube/channel. The average fluxes are defined per unit surface area of the tube/channel.

1.5.2 Correlations

Based on the driving force for the fluid velocity, heat transfer applications can be classified as *forced convection* and *natural convection* heat transfer. In natural convection heat transfer, the flow velocity is driven by the buoyancy forces due to the density variations generated by heating, and so the flow velocity depends on the temperature variations in the fluid. In this case, the Nusselt number is a function of the Grashof and Schmidt numbers, defined in equations **??** and **??**.

Natural convection mass transfer, where flow is driven by density differences due to concentration variations, is comparitively rare, though correlations similar to those for heat transfer can be derived. In forced convection heat/mass transfer, the flow velocity is generated by external means, and is independent of temperature/concentration variations in the fluid to a good approximation, though changes in viscosity due to temperature variations are often incorporated. Empirical correlations for forced convection heat/mass transfer usually relate the dimensionless fluxes (Nusselt and Sherwood number) the Reynolds number (ratio of convection and momentum diffusion) and the Schmidt number (ratio of momentum and mass diffusion).

Forced convection heat/mass transfer

Correlations for forced convection heat transfer can be classified into three broad categories.

1. **Low Peclet number, diffusion dominated**. Convective effects are neglected in the first approximation, and so the fluxes are independent of the flow velocity. If the fluxes are independent of the flow velocity and the fluid flow properties (density and viscosity), the Nusselt and Sherwood numbers do not depend on the Peclet or Reynolds numbers, and are constants from dimensional analysis. (It might be useful to refer back to the relations between dimensionless groups, equations **??** and **??**, to appreciate this). For the specific case of the heat/mass transfer from a sphere, it is shown later in chapter **??** that the Nusselt and Sherwood numbers are equal to 2 when convection is completely neglected.

   When the correction to the transport rated due to convection are considered in the limit of low Peclet number, it is shown in chapter **??** that the correlations for the Nusselt and Sherwood numbers for the flow around objects are of the form,

   $$\mathrm{Nu} = \mathrm{Nu}_0 + (1/2)\mathrm{Nu}_0^2\mathrm{Pe} + \dots \tag{1.57}$$

   where Nu_0 is the constant value of the Nusselt number when flow is neglected, and the term proportional to Pe in the above equation is the first correction due to convective effects.

   In the case of flow through tubes and channels, convection is necessarily important in order to transport heat/mass along the length of the tube. Therefore, the diffusion-dominated regime is not of interest.

2. High Peclet number. Naively, we might expect that the transport rates are independent of the diffusivities in the limit of high Peclet numbers, because convective effects dominate. However, an important concept which we will emphasise in this course is that even when the Peclet number is large, diffusion cannot be neglected. This is convective transport takes place only in the direction of fluid flow. At the surface of suspended particles or pipe walls, fluid flows in the direction tangential to the surface. Transport of

mass/heat across the surface cannot take place due to convection, because there is no fluid flow across the surface. Therefore, transport across the surface has to take place only due to molecular diffusion, and consequently diffusivity cannot be neglected even at high Peclet number.

The transport rates at high Peclet number also depend on the fluid flow characteristics. There are two distinct fluid flow regimes, in pipes and channels, the laminar flow regime with smooth streamlines at relatively low Reynolds number, and the turbulent flow regime with large velocity fluctuations at relatively high Reynolds number. In a pipe flow, the transition between the two regimes takes place at a Reynolds number of about 2100. In the flow past a flat plate, the Reynolds number is defined as $Re = (\rho U L/\mu)$ based on the free-stream velocity U and the length of the plate L. In this case, the transition from laminar to turbulent flow takes place at a Reynolds number of about 5×10^5. For the flow around a spherical particle, the flow is laminar when the Reynolds number based on the free-stream velocity and particle diameter is less than about 10. The laminar flow is characterised by smooth and axisymmetric streamlines all around the particle. When the Reynolds number increases beyond about 10, there is separation of the streamlines from the rear of the object, and formation of a wake at the rear.

(a) **Laminar flow:** The laminar flow regime is characterised by smooth streamlines, and transport across these streamlines takes place only due to molecular diffusion. Therefore, the transport rates will depend only on the flow velocity and the molecular diffusion coefficient. The Sherwood and Nusselt numbers can be expressed in terms of the Peclet number alone, though they are often written in terms of the Reynolds and Prandtl/Schmidt numbers. For the heat/mass transfer to a spherical particle, we shall see in chapter **??** that the Nusselt/Sherwood numbers depend on the one-third power of the Peclet number,

$$(\mathrm{Nu}, \mathrm{Sh}) = 1.240\mathrm{Pe}^{1/3} = 1.240\mathrm{Re}^{1/3}(\mathrm{Pr,Sc})^{1/3} \tag{1.58}$$

The above correlation is valid for a spherical particle, for which the no-slip boundary condition (fluid velocity at the surface is equal to the surface velocity) is applied at the surface. For particles of other shapes, the Nusselt and Sherwood numbers are still proportional to the one-third power of the Peclet number, but the constant is different from that in equation **??**.

In the case of transfer to a spherical bubble, for which there could be tangential flow at the surface but the shear stress at the surface is zero, it is shown in chapter **??** that the Nusselt/Sherwood number are proportional to the square root of the Peclet number,

$$(\mathrm{Nu}, \mathrm{Sh}) = ...\mathrm{Pe}^{1/2} = ...\mathrm{Re}^{1/2}(\mathrm{Pr,Sc})^{1/2} \tag{1.59}$$

The Nusselt and Sherwood numbers are proportional to the square root of the Peclet number for non-spherical bubbles as well, but the constant in the correlation is different from that in equation **??**.

Similar correlations can be derived for the Nusselt and Sherwood numbers for the flow past a flat plate. For the laminar flow past a solid surface with a no-slip boundary condition at the surface at high Peclet number, the heat/mass transfer is confined to a thin 'boundary layer' adjacent to the surface. It is shown, in chapter **??**, that the correlation for the Nusselt/Sherwood numbers are, At high Reynolds number, the correlations depend on the relative

$$(\mathrm{Nu}, \mathrm{Sh}) = 1.332\mathrm{Pe}^{1/3} = 1.332\mathrm{Re}^{1/3}(\mathrm{Pr,Sc})^{1/3} \tag{1.60}$$

where $\mathrm{Pe} = (\dot{\gamma}L^2/\mathcal{D})$, where $\dot{\gamma}$ is the strain rate (du_x/dy in figure **??**). For transport to a falling liquid film from the gas phase, we show in chapter **??** that the Nusselt/Sherwood numbers are,

$$(\mathrm{Nu}, \mathrm{Sh}) = ...\mathrm{Pe}^{1/2} = ...\mathrm{Re}^{1/2}(\mathrm{Pr,Sc})^{1/2} \tag{1.61}$$

where the Peclet number $\mathrm{Pe} = (UL/\mathcal{D})$, where U is the velocity at the surface of the liquid film. As we discuss in chapter **??**, this correlation is valid only when the penetration of heat/solute into the film is much smaller than the height of the film H.

For a laminar flow in pipes with Reynolds number less than 2100, the correlation for the Sherwood number

$$\mathrm{Sh} = 1.86\mathrm{Pe}^{1/3}(D/L)^{1/3} = 1.86\mathrm{Re}^{1/3}\mathrm{Pr}^{1/3}(D/L)^{1/3} \tag{1.62}$$

This correlation is valid for relatively short pipes with (L/D) less than about 10, and for $(\mathrm{RePr}(D/L)) > 10$. As we shall see in chapter **??**, the restriction on (D/L) comes about from the requirement that the 'penetration depth' for heat/mass transport from or to the wall should be smaller than the pipe diameter for this correlation to be valid. For longer pipes, the temperature/concentration gets equalised across the cross-section of the pipe, resulting in a decrease in the transport rates. In the correlation for the Nusselt number, a correction factor due to the variation of viscosity in the pipe is usually added.

$$\mathrm{Nu} = 1.86\mathrm{Re}^{1/3}\mathrm{Pr}^{1/3}(D/L)^{1/3}(\mu/\mu_w)^{0.14} \tag{1.63}$$

where μ_w is the viscosity at the wall temperature and μ is the viscosity at the average fluid temperature in the pipe.

(b) **Turbulent flow:** In turbulent flows, there are large velocity fluctuations, and there is significant cross-stream motion of fluids in the form of correlated parcels of fluids called eddies. Cross-stream transport in turbulent flow stakes place primarily by turbulent eddies, and

so the transport rates are much higher than those in laminar flows. However, the fluctuations decrease as the surface of a particle or the wall of the tube is approached, because the fluid cannot penetrate the solid surface. Therefore, transport very close to the surface takes place by molecular diffusion. Due to the combination of eddy diffusion and molecular diffusion, the Nusselt and Sherwood numbers depend on both the Peclet and the Reynolds numbers in turbulent flow.

For the turbulent flow through a pipe, the 'Sider-Tate' correlation is commonly used for the Nusselt Sherwood numbers,

$$\mathrm{Nu} = 0.026\mathrm{Re}^{0.8}\mathrm{Pr}^{1/3}(\mu/\mu_w)^{0.14} \tag{1.64}$$

The equivalent correlation for the Sherwood number, without the viscosity correction, is,

$$\mathrm{Sh} = 0.026\mathrm{Re}^{0.8}\mathrm{Sc}^{1/3} \tag{1.65}$$

Natural convection heat transfer

Here, convection takes place due to unstable density variations in a gravitational field, and these density variations are caused by temperature variations. The appropriate dimensionless groups are the Prandtl number and the Grashof number ($\rho^2 D^3 \beta \Delta T/\mu^2$), discussed in section **??**, which is the Reynolds number based on the convection velocity. The flow due to natural convection is laminar when the product (GrPr) is less than about 10^9, whereas the flow becomes turbulent for $(\mathrm{GrPr}) > 10^9$.

Empirical correlations are available for the convection due a single heated sphere in a large body of fluid,

$$\mathrm{Nu} = 2 + 0.59(\mathrm{GrPr})^{1/4} \tag{1.66}$$

to a long heated cylinder in an infinite fluid for $\mathrm{GrPr} > 10^4$,

$$\mathrm{Nu} = 0.518(\mathrm{GrPr})^{1/4} \tag{1.67}$$

for vertical plates suspended in air for $10^4 < \mathrm{GrPr} < 10^9$,

$$\mathrm{Nu} = 0.59(\mathrm{GrPr})^{1/4} \tag{1.68}$$

These correlations cannot be used for $\mathrm{GrPr} > 10^9$, because the flow becomes turbulent.

In chapter **??**, we shall derive expressions for the Nusselt number in the natural convection due to a heated object. There are two regimes depending on the Prandtl number, which is the ratio of momentum and thermal diffusivity. Momentum diffusion is slow compared to thermal diffusion in the limit $\mathrm{Pr} \ll 1$, and so the fluid flow is confined to a thinner region when compared to the temperature disturbance. In this case, the Nusselt number is given by,

$$\mathrm{Nu} \propto \mathrm{Pr}^{1/2}\mathrm{Gr}^{1/4} \tag{1.69}$$

At high Prandtl number, momentum diffusion is fast compared to thermal diffusion. Due to this the spatial extent of the temperature disturbance is comparable to that of the momentum disturbance, and the Nusselt number is,

$$\mathrm{Nu} \propto \mathrm{Pr}^{1/4}\mathrm{Gr}^{1/4} \tag{1.70}$$

These scaling relationships hold for particles suspended in a fluid (where the length scale is the particle size), for long horizontal tubes (where the length scale is the tube diameter) and for vertical plates (where the length scale is the plate height) In all cases, the proportionality constant depends on the detailed shape of the object. All of these correlations are applicable only for $(\mathrm{GrPr}) < 10^9$ in the laminar regime.

A simpler situation is the heat (or mass) transfer from an infinite flat plate. The configuration is as shown in figure **??**. The plate is of infinite extent in the z plane (perpendicular to the direction of flow), and has length L in the x direction. The fluid flow is in the x direction. There is a no-slip boundary condition for the fluid velocity field at the surface of the plate. The temperature in the fluid far from the plate is T_∞, while that at the surface of the plate is T_0. In the equivalent mass transfer problem, the concentration far from the plate is c_∞, while the concentration at the surface of the plate is c_0.

Since the plate is of infinite extent in the z direction, it is appropriate to define the heat transfer rate per unit length in the z direction per unit time. The Nusselt number in this case is $(qL/k(T_0 - T_\infty))$, since L is the appropriate length scale, while the Sherwood number for the case of mass transfer is $(jL/k(c_0 - c_\infty))$. The Reynolds number is $(\rho UL/\mu)$, while the Prandtl and Schmidt numbers are $(c_p\mu/k)$ and $(\mu/\rho\mathcal{D})$ respectively. When the Reynolds number is small, the correlations are, magnitudes or the Reynolds number and the Prandtl or Schmidt numbers. When the flow is in the laminar regime and the Prandtl and Schmidt numbers are small (momentum diffusion is slow compared to mass and heat diffusion), the correlations are,

$$\mathrm{Nu} = 1.128\mathrm{Re}^{1/2}\mathrm{Pr}^{1/2} \tag{1.71}$$

$$\mathrm{Sh} = 1.128\mathrm{Re}^{1/2}\mathrm{Sc}^{1/2} \tag{1.72}$$

At high Reynolds number for a laminar flow, when the Prandtl and Schmidt numbers are large (momentum diffusion is fast compared to mass and heat diffusion), the correlations are,

$$\mathrm{Nu} = 0.667\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3} \tag{1.73}$$

$$\mathrm{Sh} = 0.667\mathrm{Re}^{1/2}\mathrm{Sc}^{1/3} \tag{1.74}$$

1.6 Correlations for momentum transfer

1.6.1 Non-dimensional flux:

The non-dimensional flux in mass/heat transfer were obtained by dividing the flux by the diffusive scales, $(\mathcal{D}\Delta c/L)$ and $(\mathcal{D}_H\Delta e/L)$ respectively. Analogously,

the non-dimensional momentum flux would be defined by scaling the shear stress by $(\nu U/L)$, where ν is the kinematic viscosity. However, conventionally, the non-dimensional fluxes have been defined by scaling the momentum flux by the inertial (convective) scale, using the density and the velocity of the fluid. Using dimensional analysis, it is easy to see that the non-dimensional stress (momentum flux) has to be scaled by (ρU^2) in order to obtain a dimensionless flux. Different variations of this scaled stress are defined for the flow in channels and tubes and for the flow around particles.

For the flow around suspended particles, the 'drag coefficient' c_D is defined as,

$$c_D = \frac{F_D/A_p}{(\rho U^2/2)} \tag{1.75}$$

where F_D is the drag force, A_p is the projected area of the object perpendicular to the flow, and $(\rho U^2/2)$ is the kinetic energy of the undisturbed flow per unit volume. To within a numerical constant, the drag coefficient is equivalent to scaling the force by the inertial scale $(\rho U^2 R^2)$ in section **??**. In section **??**, we had followed the heat/mass transfer convention of scaling the force by the diffusive (viscous) force scale, to obtain $(F_D/(\mu U D))$; it is easy to see that the latter is equal to $\mathrm{Re}c_D$, the Reynolds number times the drag coefficient.

The drag coefficient for the flow past a flat plate is defined in a similar manner. For a plate of length L along the flow direction and width W perpendicular to the flow direction, the average stress is the total force exerted by the fluid divided by the area of the plate (LW), where L is the length and W is the width of the plate. The drag coefficient is then defined as the ratio of the stress and $(\rho U^2/2)$, where U is the fluid velocity far from the plate,

$$c_D = \frac{(F_D/LW)}{(\rho U^2/2)} \tag{1.76}$$

For flow through channels and pipes, the scaled momentum flux is expressed in terms of the 'friction factor'. The friction factor is usually defined in terms of the pressure difference across the pipe required for driving the fluid flow, and so we briefly discuss the relation between the pressure difference and the wall shear stress before returning to the definition of the friction factor. The force on the fluid due to the shear stress at the surface is exactly balanced by the force due to the difference in pressure at the two ends of the pipe, and therefore,

$$\tau_w L l_p = (-\Delta p) A_{cs} \tag{1.77}$$

where l_p is the perimeter of the coudiut, and A_{cs} is the cross-sectional area, and τ_w is the wall shear stress. From equation 1.77, the shear stress is given by,

$$\tau_w = \frac{-\Delta p}{L}\frac{A_{cs}}{l_p} = \frac{-\Delta p}{L} r_H \tag{1.78}$$

where r_H, the hydraulic diameter, is the ratio of the cross-sectional area and the wetted perimeter. Note that for a pipe of circular cross section, $r_H =$

$(D/4)$, where D is the pipe diameter. It is conventinal to define the 'equivalent diameter' $D = 4r_H$, so that the equivalent diameter and pipe diameter are equal for a pipe of circular cross section. Thus, the wall shear stress can be written in terms of the equivalent diameter as,

$$\tau_w = \frac{-\Delta p}{L}\frac{D}{4} \tag{1.79}$$

The friction factor is then written with momentum flux in the numerator, and the product of the difference in momentum density between the wall and center of the pipe (ρU) and the convective scale (UD) (instead of the momentum diffusivity) in the denominator

$$f = \frac{2\tau_w}{\rho U^2} = \frac{\Delta p}{L}\frac{D}{2\rho U^2} \tag{1.80}$$

The factor of 2 in the numerator on the right side of the above equation is a convention adopted in literature.

The friction factor is also used for the flow through packed columns, where the fluid flows in in the gaps between densely packed particles. Here, the friction factor is defined as,

$$f = \frac{\Delta p}{L}\frac{D_p}{2\rho U_0^2} \tag{1.81}$$

where D_p is the particle diameter, and U_0 is the 'superficial velocity', which is the average velocity of the fluid if it were to flow through a column of the same diameter with no particles. Note that the friction factor is now defined in terms of the particle diameter and not the column diameter. This is a natural definition because most of the drag force on the fluid is exerted by the particles, and not by the walls of the column. The Reynolds number is also defined based on the superficial velocity and the particle diameter, $\mathrm{Re} = (\rho U_0 D_p/\mu)$.

1.6.2 Correlations

In the limit of low Reynolds number, and the stress is purely viscous, and the pressure drop per unit length in a pipe should be independent of the fluid density. From dimensional analysis, the pressure drop per unit length can be written as,

$$\frac{\Delta p}{\Delta L} = \frac{2c\mu U}{D^2} \tag{1.82}$$

where c is a constant of $O(1)$. The friction factor is then given by,

$$f = \frac{c\mu}{\rho U D} = \frac{c}{\mathrm{Re}} \tag{1.83}$$

Thus, the friction factor is proportional to the inverse of the Reynolds number in the limit of low Reynolds number. The constant c is equal to 16 for a pipe of circular cross section.

When the Reynolds number exceeds 2100 in a pipe flow, there is a transition from laminar to turbulent flow, and the friction factor is much larger than that for a laminar flow due to the efficient cross-stream transport of momentum by cross-stream eddies. The friction factor is usually expressed graphically in the form of a Moody diagram, which shows the log of the friction factor versus the log of the Reynolds number. Plotted this way, the friction factor curve is a line with slope -1 in the laminar regime. There is a transition to a turbulent flow at a Reynolds number of 2100, indicated by a discontinuous change in the friction factor. The friction factor is also affected by wall roughness in the turbulent regime, as shown in the Moody plot.

The motion of a spherical particle in a fluid was analysed using dimensional analysis in section **??**. The drag coefficient defined in equation **??** is the drag force divided by $(\rho U^2(\pi R^2)/2)$ which is, to within a multiplicative constant, the drag force scaled by an inertial scale. As discussed in section **??**, the drag force is proportional to (μRU) in the limit of low Reynolds number, and therefore the drag coefficient is proportional to the inverse of the Reynolds number. For the flow past a spherical particle, it is shown in chapter **??** that the drag force is exactly equal to $(6\pi\mu RU)$, and therefore the drag coefficient is given by,

$$c_D = \frac{24}{\mathrm{Re}} \tag{1.84}$$

For particles of other shapes, the drag coefficient is proportional to the inverse of the Reynolds number, but the constant of proportionality depends on the shape of the object.

In the limit of high Reynolds number, the drag force depends on the shape of the object. For 'bluff bodies' such as spherical particles, we shall see in chapter **??** that an important flow characteristic is the separation of the streamlines from the surface of the object at the rear, and the formation of wake. The pressure in the wake at the rear is small compared to the upstream hemisphere at the front. The drag force, called 'form drag', is primarily due to the difference in pressure between the upstream and downstream sides. This is in contrast to the 'wall drag' in the pipe flow (which we just discussed) which is due to wall friction. Since the upstream pressure due to the fluid impact on bluff bodies is inertial, the pressure scales as ρU^2, and the drag coefficient approaches a constant in the high Reynolds number limit.

In slender bodies such as airfoils, the shape of the object is designed to shift the boundary layer separation to the rear end of the object, and to reduce form drag. Due to this, there is either no wake or a small wake region at the rear, and the drag coefficient is significantly lower. In the limiting case of the flow past a thin flat plate, we shall see in chapter **??** that the drag coefficient decreases as the inverse square root of the Reynolds number in the high Reynolds number limit,

$$c_D = ...\mathrm{Re}^{-1/2} \tag{1.85}$$

where $\mathrm{Re} = (\rho UL/\mu)$ is the Reynolds number based on the length of the plate and the free stream fluid velocity far from the plate.

Next, we determine the relation between friction factor and Reynolds number in packed beds. The flow through a packed bed can be analysed by approximating the interstices between the particles in the bed as tubes of effective diameter R and length equal to the column length L. For a laminar flow, the relation between the pressure drop and the avearge velocity of the fluid through the tubes U is,

$$\frac{\Delta p}{L} = \tag{1.86}$$

1.7 Scale up

In designing new processes, it is not sufficient to study the the process on the laboratory scale, but to also study the exact industrial set up on a small scale before building a larger scale industrial apparatus. Dimensional analysis plays a very important role in industrial scale up. Scaling up cannot be done by multiplying all parameters by a given factor, but the dimensionless groups have to be kept a constant while scaling up. This will be illustrated using the example of a stirred tank reactor. In this, the fluid is stirred using an impeller of a certain shape, and the impeller is to be designed so that optimum mixing is achieved for minimum power. For a given impeller shape, it is necessary to estimate the power consumption for stirring at a given frequency f. The power consumption will, in general, depend on the shape and dimension of the impeller, as well as the vessel, as well as other details such as baffles, etc. If we keep the relative ratios of the lengths of the impeller, vessel, baffles, etc. a constant, then there is only one length scale in the problem, which we will consider the impeller diameter D. In addition, the power can also depend on the density of the fluid, ρ, the fluid viscosity μ, and the frequency of rotation f. An additional dependence arises on the acceleration due to gravity g. This is because sometimes during stirring, the interface of the fluid raises at the edges and lowers in the middle. This upward motion due to centrifugal forces is balanced by a downward force due to gravity, and so gravity could also be an important factor. This is because it is important to ensure that the dimensions of the interface (curvature, extent of depression at the center) are also in the same proportion as the impeller diameter. When considering the interface, the surface tension (normally denoted by the symbol γ) is also a relevant parameter. the surface tension has dimensions of force/length of energy/area, i. e. $\mathcal{MT}^{-2}$. The relevant parameters with their dimensional dependences are shown in table 1.7

The dimensionless variables derived above have the following physical interpretations. The 'power number' $(P/f^3D^5\rho)$ is the dimensionless group which involves the dependent variable, the power, which has to be determined as a function of all the other independent variables. This gives the ratio of the power required to the work done by centrifugal forces. On the right side, the 'Reynolds number' $(\rho D^2 f/\mu)$ is the ratio of centrifugal forces and viscous forces, or the ratio of convection and diffusion. The dimensionless group (f^2D/g) is the Froude number, which gives the ratio of centrifugal and gravitational forces.

Parameter	Dimension
Power (P)	$\mathcal{M}\mathcal{L}^2\mathcal{T}^{-3}$
Frequency (f)	$\mathcal{T}^{-1}$
Diameter (D)	$\mathcal{L}$
Density (ρ)	$\mathcal{M}\mathcal{L}^{-3}$
Viscosity (μ)	$\mathcal{M}\mathcal{L}^{-1}\mathcal{T}^{-1}$
Gravity (g)	$\mathcal{L}\mathcal{T}^{-2}$
Surface tension (γ)	$\mathcal{M}\mathcal{T}^{-2}$

Table 1.5: Relevant quantaties and their dimensions for the calculation of the power required for the impeller in a reactor.

The dimensionless group $(\rho f^2 D^3/\gamma)$ is the Weber number, which is the ratio of centrifugal and surface tension forces.

Since there are seven variables and three dimensions, it is possible to create four dimensionless groups. Let us assume these four contain P, μ, g and γ expressed in terms of the other variables. Then, the four groups are $(P/f^3D^5\rho)$, $(fD^2\rho/\mu)$, (f^2D/g) and $(\rho f^2D^3/\gamma)$. Therefore, the expression for the power has to have the form

$$\frac{P}{f^3D^5\rho} = \Phi\left(\frac{\rho D^2 f}{\mu}, \frac{f^2 D}{g}, \frac{\rho f^2 D^3}{\gamma}\right) \tag{1.87}$$

It is instructive to determine the order of magnitudes of the different dimensionless groups in the problem. The density of the liquid is usually of the order or 10^3 kg/m^3, the viscosity of a very viscous fluid such as a polymer melt could be as high as 1 kg/m/s, and the surface tension of a liquid-gas interface is, at maximum, 0.1 kg/s^2. If the frequency is of the order of 1-10 rev/s, the Froude number $(f^2D/g) \approx 0.1-10$, indicating that both centrifugal and gravitational forces are important in the present problem. The Weber number $(\rho f^2D^3/\gamma) \approx 10^4 - 10^6$, which is large, indicating that the surface tension effects are small when compared to inertial effects. Therefore, the effect of surface tension can be neglected in the present application. The Reynolds number $(\rho D^2 f/\mu) \approx 10^3 - 10^4$, which is large. Therefore, it might naively be expected that viscous effects can be neglected in comparison to inertial effects. However, as we shall see in the analysis of boundary layers, diffusion can not be neglected, because it is diffusive transport which is responsible for the transport of mass, momentum and energy at the bounding surfaces of the fluid. With the neglect of surface tension effects, the relation 1.87 reduces to,

$$\frac{P}{f^3D^5\rho} = \Phi\left(\frac{\rho D^2 f}{\mu}, \frac{f^2 D}{g}\right) \tag{1.88}$$

In a scale up, the dimensionless numbers have to be kept a constant. For example, we are interested in designing a reactor with an impeller diameter of $1m$ with a revolution of $10rev/s$, and the fluid in the reactor is water with density $1000kg/m^3$ and viscosity $10^{-3}kg/m/s$. In order to determine the performance, we design a smaller reactor with an impeller of size $10cm$. What is the fluid

that should be used, and what is the speed at which the reactor should operate? The speed of rotation can be determined from the consideration that the Froude number has to be a constant. If quantities for the big reactor are denoted with the subscript b and those for the small reactor are denoted by the subscript s, then for the Froude number to be a constant, we require

$$\frac{f_b^2 D_b}{g} = \frac{f_s^2 D_s}{g} \tag{1.89}$$

Substituting the dimensions and the frequency of the big reactor, we get the impeller speed of the small reactor as $316 rev/s$. The choice of fluid to be used in the small reactor is determined by the condition that the Reynolds number has to be a constant.

$$\frac{\rho_b f_b D_b^2}{\mu_b} = \frac{\rho_s f_s D_s^2}{\mu_s} \tag{1.90}$$

Relating the frequency and diameters of the two reactors, we get

$$\frac{\rho_s}{\mu_s} = 31.6 \frac{\rho_b}{\mu_b} \tag{1.91}$$

Finally since the Reynolds number and the Froude number are kept a constant between the two configurations, the Power number is also a constant

$$\frac{P_b}{f_b^3 D_b^5 \rho_b} = \frac{P_s}{f_s^3 D_s^5 \rho_s} \tag{1.92}$$

Therefore, the ratio of the power required in the two configurations is

$$\frac{P_b}{P_s} = 3160 \frac{\rho_b}{\rho_s} \tag{1.93}$$

From the power requirement of the small reactor, the power estimate for the big reactor can be obtained using the above relation.

The power number is used in reactors with impellers in order to express the power requried as a function of the frequency and the diameter of the impeller. The power number is defined as

$$\mathrm{Po} = \frac{P}{\rho \omega^3 D^4} \tag{1.94}$$

where P is the power consumed by the impeller, ω is the frequency of rotation and D is the impeller diameter.

Exercises

1. The dimensionless groups for the heat flux in a heat exchanger were determined assuming that there is no inter-conversion between mechanical and thermal energy. If mechanical energy can be converted to thermal energy, there would be one additional dimensionless group which would be relevant for the heat flux. What is this dimensionless group, and what is its significance?

2. The Maxwell equations in electrodynamics are,

$$\nabla.\mathbf{E} = (\rho_s/\epsilon_0) \tag{1.95}$$

$$\nabla.\mathbf{B} = 0 \tag{1.96}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{1.97}$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t} \tag{1.98}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic field vectors, ρ_s is the charge density, ϵ_0 and μ_0 are the permittivity and permeability of free space, and $\mathbf{J}$ is the current density. The dimensions of electric field $\mathbf{E}$ is (Volt/meter), and that of charge density ρ_s is (Coulombs/ m^3), or (Amps $\times$ s / m^3), where m is meters and s is seconds. Using the above equations, **determine the dimensions of E, B**$, \epsilon_0, \mu_0$ **and J in terms the fundamental dimensions of mass, length, time and amperes**. Note that $(\partial/\partial t)$ is the partial derivative with respect to time, $\nabla.$ and $\nabla\times$ are the divergence and curl operators with dimensions of inverse length, and

$$\text{Power} = \text{Volt} \times \text{Ampere} \tag{1.99}$$

3. Use dimensional analysis to obtain the dimensionless groups in the problem of a droplet of oil placed in a water bath placed in an electric field between two plane electrodes separated by a distance L, and moving with velocity U, as shown in figure 1. Assume the density of oil and water are equal. The important dimensional parameters are,

   (a) the radius of the droplet R,

   (b) the distance between the electrodes L (assume they are of infinite extent in the $x - y$ plane),

   (c) the velocity of the droplet,

   (d) the viscosity of water,

   (e) the viscosity of oil,

   (f) the density of water/oil,

   (g) the surface tension between water and oil,

(h) the voltage difference between the two electrodes with dimension Volts,

(i) the dielectric constant of water ϵ_w, with dimension $(A^2 s^4 kg^{-1} m^{-3})$, where A is amperes, and

(j) the dielectric constant of oil ϵ_o, with dimension $(A^2 s^4 kg^{-1} m^{-3})$.

Note that the dimension of potential, Volt, is related to the dimension of current, amperes, by the relation

$$\text{Power} = \text{Volt} \times \text{Ampere} \tag{1.100}$$

How many dimensionless groups are there in the above problem? Of these, three are easily obtained as the ratio of lengths (L/R), the ratio of viscosities and the ratio of dielectric constants of water and oil. The other dimensionless groups are expressed as the ratios of stresses caused by different physical mechanisms. Identify the dimensionless groups, and the physical mechanisms.

How would the problem simplify if the distance between plates is large compared to the droplet radius?

4. It is desired to set up a spray drier for drying a solution of viscosity 0.1 kg/m/s and density 1000 kg/m^3, and containing 80 % by weight of water, into particles of diameter about 100 μm. In order to achieve this, it is necessary to design the nozzle used for ejecting the spray, the diameter of the spray drier and the hot air to be circulated through the drier. Use dimensional analysis to determine the design considerations. Due to the temperature sensitivity of the product, it is not possible to have an average difference in temperature between the droplet and the air of more than 40°C.

   This example is an instance where fluid flow is coupled with heat and mass transfer. In the spray drier, the flow through a very small nozzle breaks up the liquid into small drops, and the water in these drops is dried by the heat transferred from the air as the drops move through the air. The drops in nozzle spray driers are usually coarse, and sizes upto 100 μ m can be achieved. For finer drops, it is necessary to use a spray disk drier, where the spraying is done by a disk of about 1 ft in diameter rotating at speeds as large as 1000 rev/s. In the present example, we consider a nozzle type spray drier, since a relatively coarse size is required. The droplets are usually ejected with high velocities, as large as 0.1 - 1 m/s, and so the time required for drying the droplets is usually very small, of the order of seconds.

   The spraying process can be separated into two distinct steps,

   (a) The ejection of the drop from the nozzle. The size of the drops, and the velocity of the drop at the nozzle, are determined primarily by the

nozzle geometry, fluid properties and the flow rate of fluid through the nozzle.

(b) The subsequent drying of the drop as it passes through the air. Here, it is necessary to ensure that the spray drier has a sufficient radius that the drop is completely dried before it hits the wall of the drier. Important for determining this are the velocity with which the drop leaves the nozzle, the heat transfer rate for transporting heat from the drop to the air, the latent heat of evaporation which determines the heat required to dry the drop.

Use dimensional analysis to design the nozzle and the diameter of the spray drier.

(a) What are the relevant dimensional variables which determine the diameter of the droplet generated form the nozzle? Estimate the dimensionless groups to determine the dominant parameters.

(b) Determine the radius of the drier to ensure that the drops are completely dried before they hit the wall. First examine heat transfer considerations. What is the latent heat required to evaporate the water from the droplet? What are the quantities which determine the heat flux from the droplet? Estimate them. The latent heat of water is 2.2×10^6 J/kg, and the thermal conductivity of air is 2×10^{-2} J/m/s/^{o}C

(c) Examine mass transfer characteristics. What are the dimensionless groups for mass transfer? Estimate them.

5. **Example** The waste water generated from a chemical process has to be passed through activated carbon in order to remove organic matter by adsorption. The carbon particles are, on average, 1mm in diameter. The process of adsorption is described by a first order process, with a rate constant 1.6 s^{-1}. It is desired to treat 160 l/min of the waste water, and the maximum speed of the water through the activated carbon bed is 1 mm/s, and the porosity (void fraction) of the bed is 0.45. Design the height and the cross sectional area of the bed.