The second normal-stress difference in avalanching granular flows

N. M. Vriend and J. N. McElwaine

Department of Applied Mathematics and Theoretical Physics,

*University of Cambridge, Cambridge, CB3 0WA, UK**

Granular avalanches on an open slope form channels, where a flowing region is bounded by quasi- static levees. Current theories are inaccurate for these flows because they contain static and flowing regions at depths close to h_{stop} [1]. The dimensions of the unconfined channels and the relations between the flowing and static regions provide insight on the rheology of the flow [2]. Previous work [3] highlighted laboratory experiments of unconfined shallow granular flows featuring a curved free surface in the flowing regions, bounded by static margins exerting lateral stresses on the flow. As the velocity profile and the height of the margins are self-similar, the rheological parameters cannot be uniquely determined, since the lateral stresses and flow depth both effect the velocity.

The current study investigates the characteristics of a flowing granular material down a low-angle V-shaped inclined plane under the action of gravity. The long-wave (shallow water) approximation relates the geometry of the flow directly to the slight V-shaped angle and therefore the depth dependence of the velocity can be decoupled from the lateral stresses.

Carefully performed laboratory experiments (panel a) show that the surface of the flow, steady both in time and down the slope, has significant curvature caused by second normal-stress differ- ences, similar to those observed in non-Newtonian fluids such as granular suspensions [4]. The curved top surface may be up to 30 % higher than the margins of the flowing region. The DEM simulations (panel b) show a similar surface profile, but also reveal details about the internal flow structure featuring a down-welling region in the middle and up-welling of material along the edges. Theoretical calculations involving a new granular theory explaining surface curvature across a slope on a flat surface [5] may be easily adapted to include the V-shape geometry. Experiments and numerical simulations validate the new theoretical model relating the height and velocity of the flow directly to the V-shape angle.

*Electronic address: nv253@cam.ac.uk

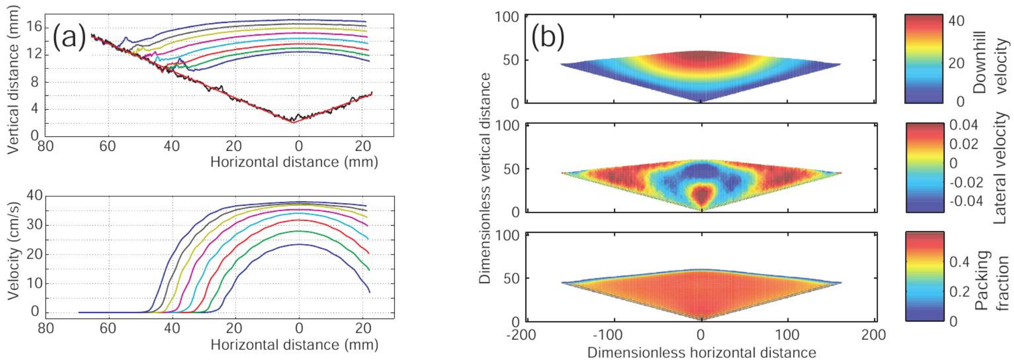

Figure 1: Steady granular flow down a V-shaped channel: (a) experiments with V-angle 11 degrees and slope angle 32 degrees. Surface heights and velocities for different flow rates, (b) velocity and density cross-sections from DEM simulations of a 15 degree V-angle on a 35 degree chute.

References

[1] G. Felix and N. Thomas, Earth Planet. Sci. Lett. **221**, 197(2004).

[2] G. MiDi, Eur. Phys. J. E **14**, 341 (2004).

[3] D. Takagi, J. McElwaine, and H. Huppert, Physical Review E p. 031306 (2011).

[4] S. B. Savage, J. Fluid Mech. **92**, 53 (1979).

[5] J. McElwaine, D. Takagi, and H. Huppert (2011), submitted to Granular Matter.