**Nonlinear analysis for a sheared granular flow: relaxation to a steady state and response around a steady state**

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Granular flow under a plane shear is the simplest and the most appropriate situation to understand *unusual* nature of granular hydrodynamics. In this talk, we present the current achievement through our stuy on the shared flow for moderate dense granular gases. This talk consists of the following three parts.

In the first part, we briefly verify the validity of a set of granular hydrodynamic equations derived from the granular kinetic theory e.g.[1] from the direct comparsion with the result of molecular dynamics simulation.[2] It is rather surprised that the granular hydrodynamics can reproduce shear band formation even if very dense region exists.

In the second part, which is the main part of this talk, we develop the weakly nonlinear anaysis of a two dimensional sheared granular flow after a uniform sheared state becomes unstable.[3] Here, we have derived the TDGL equation starting from a set of granular hydrodynamic equations. Through our analysis the appearance of a nonlinear state is characterized by a supercritical bifurcation in the dilute case but by a subcritical bifurcation for denser case.

In the last part, we discuss the response theory around a NESS of sheared granular flow based on the adiabatic approximation and the complete counting statistics.[4] Through this analysis, we demonstrate what the essential differences is between the response theory around an equilibrium state and the response theory around a NESS. It is remarkable that the response theory around a NESS depends on the protocol, i.e. the path of the changing the shear rate.

## References

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