Dissipative potentials in the mechanics of particulate media

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This paper is concerned with the usage of generalized dissipation functions or "dissipative potentials" in various theories of viscoplasticity, with special emphasis on the application to fluid-particle suspensions and granular media. As generalizations of the classical Rayleigh dissipation function, such functions have been employed in plasticity theories at least as far back as the early work of Melan [3, 5], and have been generalized to Cosserat continua [4]. A cursory survey is given of select applications of such models to the plasticity of granular materials.

Physical justifications for the existence of dissipative potentials are often based on the as- sumption of maximum dissipation (or entropy generation) [6], which is plausible for Stokesian suspensions but less than evident for granular materials generally. By contrast, in remarkable work that seems to have been largely ignored, Edelen [1, 2] has proposed a purely mathematical construct giving general force-dependent velocity (or thermodynamic "flux") as the gradient of a dissipative potential plus a non-dissipative term that represents Ziegler's "thermodynamically orthogonal" flux and the "gyroscopic" effects that are usually ruled out of Onsager-type theories.

The present work provides an elementary derivation of Edelen's results by means of conventional vector calculus, and at the same time expores the possibility of representing kinematic constraints such as granular dilatancy in terms of the non-dissipative terms in the Ziegler- Edelen formulae.

References

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