

**Local and nonlocal models for dense granular flow**

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Dense granular materials such as gravel and sand compose a family of heterogeneous media with a number of modeling challenges. While discrete element approaches have been useful and have had success in reproducing experimental results, a feasible and broadly applicable continuum law for granular flow is much desired, that can predict stress and flow fields in arbitrary geometries. This talk uses data from experiments and discrete element simulations to determine a common size-scale for RVE behavior in common granular materials, and synthesizes a meso-scale elasto-plasticity law for steady flow. The law is tested and verified in a number of geometries by comparing against known data. For reasons to be explained, the law is less sufficient in slowly sheared regions, where a verifiable nonlocal constitutive behavior can emerge. Nonlocal effects cause the actual flow to differ from the local law in several ways: (1) Slowly driven shear-bands have a non-vanishing width scaled by the particle size; (2) In the presence of a stress gradient, material undergoing stresses less than the yield criterion can still flow; (3) The flow-rate in slowly sheared regions is independent of the size of the stress, in sharp contrast with the local law. We address these issues by adding a nonlocal enhancement term to our law, which enables us to capture all of these effects. This term is inspired by theoretical work on nonlocal fluidity in the emulsions community. We demonstrate the merits of the extended law by testing its predictions against multiple DEM simulations in multiple geometries.

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