Segregation of particles in incompressible random flows: singularities, intermittency and random uncorrelated motion

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The transport of particles/droplets dispersed in turbulent flows is of crucial importance to a wide range of natural and engineering processes. In this theoretical and numerical study, we focus on the transport of heavy particles in an incompressible gas flow and exploit a Full Lagrangian method to measure the statistical properties of the particle segregation. While doing so, we are able to analyse some particular features of this ongoing process, and in particular to study the statistics of singularities in the particle concentration field and the recently observed Random Uncorrelated Motion (RUM): the velocity of particles with large inertia brought into close proximity may be strongly decorrelated not only with the flow but one with another.

In our recent work (IJzermans et al, 2009 and 2010), we have studied the segregation of heavy particles in turbulence by calculating the rate-of-compression of the particle phase in a kinematic simulation. Particles are advected by Stokes drag in a flow field composed of 200 random Fourier modes. The volume occupied by the particles centred around a position x at time t is denoted by J = det(J_{ij}), where $J_{ij} = \partial x_i x_0 / \partial x_{o,j}$, where x_0 denotes the initial position of the particle. The particle-averaged compressibility, $\zeta = d < ln|J| >= dt$, gives a measure for the change of the total volume occupied by the particle phase. Numerical results showed that the particleaveraged rate-of-compression decreases continuously if the value of the Stokes number (the dimensionless particle relaxation time) is below a threshold value, $St_c r$, indicating that the segregation of these particles continues indefinitely. We find that the probability density function of $ln|J|$, the compression, tends to a Gaussian distribution for $St \sim 1$ when $t \to \infty$. We believe the explanation for Gaussianity is similar to that for the occurrence of a Gaussian distribution of displacement (Taylor, 1922), with $\zeta'(t)$, the fluctuating value of $\zeta(t)$ about its mean. However, we find that that such PDF shows a significant skewness towards negative compression (segregation), i.e. singularities in the flow are likely to play a significant role in determining the statistics of the segregation in these long term limit

By counting events for which $|J(t)| = 0$, we can calculate the distribution of singularities over a fixed interval of time respectively for a set of St numbers. As shown in Figure 1 for $St = 1$, excluding the influence of an initial transient when no singularities are observed, the histogram that represents the discrete probability distribution is well approximated by a Poisson distribution that describes the probability of the

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occurrence of an event (singularity) in a specified time span $[0; \Delta t]$ as $\sim \lambda \Delta t = \Lambda$; λ is the rate constant for the occurrence of singularities. The Poisson process implies that starting from some initial fully mixed equilibrium distribution, the decay in the number of particles that have not experienced a singularity is $\sim exp(-\lambda t)$.

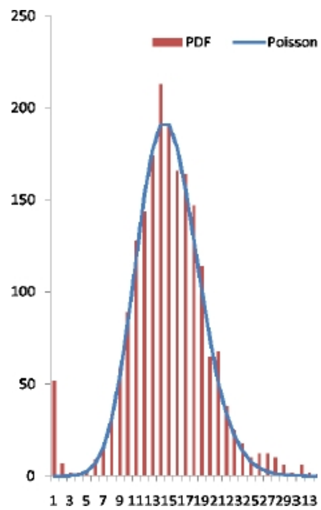

Figure 1: Comparison between theory and experimental data.

Finally, we discuss our work in relation to that of Falkovich & Pumir, 2007 and Wilkinson *et al*, 2007 and conclude that the occurrence of singularities is related to the formation of caustics and sling effect respectively, since it corresponds to the folding of the particle velocity field in phase space. We believe that RUM and singularities are intrinsically related and we are currently working to find a suitable way to demonstrate such theory from a mathematical and numerical point of view.

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