

Effect of convective transport on droplet spinodal decomposition in fluids

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The effect of convective transport on the late stage growth of droplets in the presence of sedimentation and shear flow is analyzed. The high Peclet number limit ($UR/D \gg 1$) is considered, where U is the characteristic velocity, R is the radius of the droplet, and D is the diffusion coefficient. The growth of the droplet depends on the boundary condition for the fluid velocity at the droplet interface, and two types of boundary conditions are considered. For a rigid interface, which corresponds to the interface between a solid and a fluid, the tangential velocity is zero and the normal velocity is equal to the velocity of the surface. For a mobile interface, which corresponds to an interface between two fluids, the tangential and normal velocities are continuous. These results indicate that the scaling relations for the critical radius are $R_c(t) \propto t^{(1/2)}$ for a sedimenting droplet with a rigid interface, $R_c(t) \propto t^{(2/3)}$ for a sedimenting droplet with a mobile interface, $R_c(t) \propto t^{(3/7)}$ for a droplet with a rigid interface in a simple shear flow, and $R_c(t) \propto t^{(1/2)}$ for a droplet with a mobile interface in a simple shear flow. The rate of droplet growth is enhanced by a factor of $Pe^{(1/3)}$ for rigid interfaces and $Pe^{(1/2)}$ for mobile interfaces. © 1998 American Institute of Physics. [S0021-9606(98)51030-6]

I. INTRODUCTION

It has recently been realized¹⁻³ that the dynamics of spinodal decomposition and droplet coarsening in fluids could be very different from that in solid alloys. The coarsening process in solid alloys proceeds by diffusion of the molecules along a solid lattice, but in a fluid convective transport could also enhance the coarsening process. The dynamics of spinodal decomposition could be distinguished into two types—the coarsening due to the motion of random interfaces forming a bicontinuous pattern in a symmetric quench,⁴⁻⁶ and the growth of droplets in an off-symmetric quench. The coarsening of droplets is considered to be due to two possible mechanisms:

- (1) the diffusion mechanism,^{8,9} which involves the growth of droplets with radius larger than a critical value and the shrinking of droplets with radius smaller than this value due to diffusion of the solute through the matrix;
- (2) the coagulation mechanism,⁷ where droplets undergoing Brownian motion collide and coalesce to give a larger droplet.

The former mechanism is observed when the droplet density is relatively low so that there are no collisions between droplets, while the latter is observed when the droplet density is higher. In both cases, the mean radius of the droplets obeys a scaling law of the form $R(t) \propto t^{1/3}$. The scaling law is the same as that for the Lifshitz–Slyozov^{8,9} theory for alloys, but it has experimentally been observed that the rate of growth of the average radius could be much larger than that in an alloy. In the present analysis, the effect of droplet motion on the diffusion mechanism is examined.

There have been many experimental studies on the growth of droplets in binary fluids. The experiments of Wong and Knobler¹⁰ indicate that the characteristic length in

the system $l(t)$ depends on time as $l(t) \propto t^\alpha$, where α is reported to be close to 0.3 if the volume fraction of the solute is small (droplet coarsening) and close to 1.0 when the volume fraction is large (hydrodynamic coarsening). Though the exponent for the droplet coarsening is close to that of the Lifshitz–Slyozov theory, it has been noticed that the rate of coarsening is larger than that predicted by the Lifshitz–Slyozov theory.

The motion of a droplet, either due to sedimentation or due to an externally imposed shear, could cause an enhancement of the transport rates of the solute at the droplet interface. In particular, when the convective effects are strong compared to the diffusive effects, the concentration field around a droplet of radius R decays to the matrix concentration within a distance small compared to the droplet radius. This could lead to larger concentration gradients, and consequently faster transport of the solute from the matrix to the droplet, resulting in faster growth. The acceleration of droplet growth due to an externally imposed shear flow has been observed experimentally.^{11,12} In the next section, scaling arguments are presented to determine the boundary layer thickness around a droplet. Two types of boundary conditions for the velocity at the surface of a droplet are considered. A droplet with a “rigid” surface is similar to a rigid sphere which has zero tangential velocity at the surface, and this boundary condition is appropriate for systems with surface active agents which prevent compression of the surface. A droplet with a “mobile” surface is similar to a liquid drop with a finite velocity at the surface, and the velocity field is determined within and outside the droplet by matching the tangential velocity and stress at the surface. The flux of the minority phase at the surface in the presence of strong convective effects is calculated, and the scaling law for the change in the critical radius as a function of time is determined. In addition, the size distribution of the droplets is also

determined and the mean radius and the number of droplets per unit volume are calculated as a function of time.

II. ANALYSIS

The system consists of a binary fluid mixture in the late stage of droplet growth in a matrix where the supersaturation is small. The concentration of the solute in the droplet is considered to be the equilibrium concentration of the droplet phase, while the matrix has a concentration c_s at a large distance from the droplet.

It is useful to estimate the range of droplet sizes for which convective effects could be important. For droplets sedimenting in a fluid, the sedimentation velocity is $(2\Delta\rho gR^2/9\mu)$, where $\Delta\rho$ is the difference in the densities of the droplet and matrix and μ is the shear viscosity. The ratio of convective and diffusive transport is given by the dimensionless ‘‘Peclet’’ number $Pe=(UR/D)$, where U is the characteristic droplet velocity, R is the radius of the droplet, and D is the diffusion coefficient. Using typical values of $\Delta\rho\sim 10^2\text{ kg m}^{-3}$, $\mu\sim 10^{-3}\text{ kg m}^{-1}\text{ s}^{-1}$, and $D\sim 10^{-9}\text{ m}^2\text{ s}^{-1}$, the Peclet number is $2\times 10^{14}R^3$, where the radius R is measured in meters. This indicates that the Peclet number is large for droplets larger than about $10\ \mu\text{m}$ settling under the effect of gravity. For a sheared suspension, the Peclet number scales as $\Gamma R^2/D$, where Γ is the shear rate. For a typical shear rate of 1 s^{-1} , the Peclet number is large for droplet sizes greater than about $10\ \mu\text{m}$. In these cases, the Lifshitz–Slyozov theory for late stage growth would not be applicable, and it is necessary to determine the growth rate in the presence of convection and diffusion.

In accordance with the classical Lifshitz–Slyozov theory, the chemical potential of the matrix at the surface of the droplet is considered to be the sum of the chemical potential of the droplet material and a correction due to surface tension. The concentration in the matrix at the surface of the droplet material c_R is

$$c_R = c_\infty \left(1 + \frac{2\sigma v}{TR} \right), \quad (1)$$

where c_∞ is the concentration that is at equilibrium with a flat interface, σ is the surface tension and v is the molecular volume. In the Lifshitz–Slyozov theory, the concentration field in the vicinity of the droplet is assumed to respond instantaneously to changes in the droplet radius, so that the concentration field is obtained by the solution of the steady state diffusion equation. In the presence of droplet motion, however, there is a transport of material due to convection and diffusion, and it is necessary to solve the steady state convection–diffusion equation to obtain the concentration field around a droplet.

The velocity field around a droplet in uniform motion is governed by the steady state Navier–Stokes equations. In addition, the inertial forces are small compared to the viscous forces for situations of practical interest. The Reynolds number, $(\rho UR/\mu)$, which is the ratio of inertial and viscous forces, scales as $(D/\nu)Pe$, where ν is the kinematic viscosity. The diffusion coefficient D is usually about $10^{-9}\text{ m}^2\text{ s}^{-1}$ for simple fluids, but could be as low as $10^{-12}\text{ m}^2\text{ s}^{-1}$ for

complex fluids such as polymer melts. The kinematic viscosity ν is $10^{-6}\text{ m}^2\text{ s}^{-1}$ for a simple fluid such as water, but could be $10^{-3}\text{ m}^2\text{ s}^{-1}$ or higher for polymer solutions and melts. Consequently, there exists a considerable range of parameter values, $10^3\leq Pe\leq 10^6$, where the Reynolds number is small and the Peclet number is large. In this case, the fluid flow around the droplet is viscous, but the convective effects are large compared to the molecular diffusion of the solute. The scaling of the droplet growth in this regime is analyzed here.

The velocity field depends on the mobility of the surface of the droplet. Consider a droplet settling in a fluid with velocity U . If the interface is rigid, so that a no-slip boundary condition is applicable at the interface, the velocity field is¹³

$$u_r = U \cos(\theta) \left[\frac{3}{2} \frac{R}{r} - \frac{1}{2} \frac{R^3}{r^3} \right], \quad (2)$$

$$u_\theta = -U \sin(\theta) \left[\frac{3}{4} \frac{R}{r} + \frac{1}{4} \frac{R^3}{r^3} \right],$$

where u_r and u_θ are the radial and azimuthal components of the velocity field, the radius r is the distance from the center of the droplet and the azimuthal angle θ is the angle made by the radius vector with the direction of the velocity of the droplet. If the interface of the drop is mobile, the velocity field is obtained by solving the Stokes equations inside and outside the droplet, and using the continuity of tangential velocity and stress at the interface. In this case, the velocity field depends on the droplet viscosity as well as the viscosity of the fluid inside and outside the droplet

$$u_r = U \cos(\theta) \left[\frac{\Sigma}{2} \left(\frac{R}{r} - \frac{R^3}{r^3} \right) + \frac{R^3}{r^3} \right], \quad (3)$$

$$u_\theta = -U \sin(\theta) \left[\frac{\Sigma}{4} \left(\frac{R}{r} + \frac{R^3}{r^3} \right) - \frac{1}{2} \frac{R^3}{r^3} \right],$$

where $\Sigma = (3\mu_r + 2)/(\mu_r + 1)$, and μ_r is the ratio of the viscosities of the droplet fluid and the matrix fluid. For a droplet in shear flow, it is convenient to express the velocity field in indicial notation in terms of the position vector \mathbf{x} in the fluid. If the strain rate is G_{ij} at a large distance from the surface of the droplet, the velocity field in indicial notation for a rigid surface is

$$u_i = G_{ij}x_j \left(1 - \frac{R^5}{r^5} \right) + \frac{5}{2} G_{jk}x_i x_j x_k \left(\frac{R^5}{r^7} - \frac{R^3}{r^5} \right) \quad (4)$$

and for a mobile surface is

$$u_i = G_{ij}x_j \left(1 - \frac{R^5}{r^5} \frac{\mu_r}{1 + \mu_r} \right) + G_{jk}x_i x_j x_k \left(\frac{5}{2} \frac{R^5}{r^7} \frac{\mu_r}{1 + \mu_r} - \frac{R^3}{r^5} \frac{2 + 5\mu_r}{1 + \mu_r} \right). \quad (5)$$

The concentration field is obtained by solving the convection–diffusion equation,

$$\partial_t c + u_i \partial_i c = D \partial_i^2 c, \quad (6)$$

where $\partial_i \equiv (\partial/\partial x_i)$ is the gradient operator, and the velocity u_i is given by 2, 3, 4, or 5. This equation cannot be solved analytically for the present configuration, but it is possible to obtain analytical solutions in the limit where the Peclet number is large.

In the large Peclet number limit, the diffusion term in Eq. (6) is small compared to the convection term, and one might attempt to get a leading order solution by neglecting the diffusion term in the equation. However, this converts the equation from a second order differential equation to a first order differential equation, and it is not possible to satisfy the boundary conditions at infinity and at the surface of the droplet using this approximation. This is because the diffusion terms become important in a boundary layer at the surface of the droplet where the concentration gradients are large. In this case, the magnitude of the concentration and flux at the surface can be obtained using scaling arguments. Consider a spherical coordinate system where the origin is located at the center of the droplet, the radius r is the distance from the origin and the azimuthal angle θ is the angle made by the radius vector with the direction of the velocity of the droplet. Let the thickness of the boundary layer be δR , where the small parameter δ is determined from the following scaling arguments. In the boundary layer, the diffusion term in Eq. (6) scales as $D(c_s - c_R)/(\delta^2 R^2)$, where c_s is the concentration of the supersaturated matrix at a large distance from the droplet. The convective transport depends on the relative velocity between the matrix and droplet in the boundary layer, $v_i = u_i - U_i$, and the magnitude of this velocity depends on the mobility of the interface.

- (1) For a *rigid interface*, where a no slip condition is applicable, Eqs. (2) and (4) indicate that the components of the relative velocity between the droplet and fluid are $v_r \sim \delta^2 U$ and $v_\theta \sim \delta U$. Equating the convection and diffusion terms, it is found that $\delta \sim \text{Pe}^{-1/3}$. The concentration flux at the interface, which is $J = D(\partial c/\partial r)$ at $r = R$, scales as $\text{Pe}^{1/3} D(c_s - c_R)/R$.
- (2) For a *mobile interface*, where a nonzero velocity is permitted at the interface, Eqs. (3) and (5) provide the components of the relative velocity $v_r \sim \delta U$ and $v_\theta \sim U$. Equating the convection and diffusion terms, it is found that $\delta \sim \text{Pe}^{-1/2}$. The flux at the interface scales as $J \sim \text{Pe}^{1/2} D(c_s - c_R)/R$.

The scaling of the concentration flux with Peclet number for a rigid interface has been observed in experiments on mass transfer to solid particles,¹⁴ but there do not appear to have been systematic experiments done to determine the flux in droplets.

The rate of growth of the droplets can now be obtained to within a constant using the scaling relations for the fluxes at the interface. The procedure, which is very similar to that used to obtain the growth law in the classical Lifshitz–Slyozov theory, is briefly described here. The rate of change of the radius is

$$\frac{dR}{dt} = J = \frac{B}{R^\beta} \left[\Delta(t) - \frac{\alpha}{R} \right] \quad (7)$$

where $\Delta(t) = c_s - c_\infty$, $\alpha = (2\sigma v c_\infty/T)$ and the constants B and β are determined as follows.

- (1) For a *sedimenting* droplet, the settling velocity $U \sim R^2$ and $\text{Pe} \sim R^3$, and for a *rigid* interface the coefficients B and β are

$$\begin{aligned} B &= B_1 (2\Delta\rho g/9\mu)^{1/3} D^{2/3}, \\ \beta &= 0. \end{aligned} \quad (8)$$

For a sedimenting droplet with a *mobile* interface, the coefficients B and β are

$$\begin{aligned} B &= B_2 (2\Delta\rho g/9\mu)^{1/2} D^{1/2}, \\ \beta &= -1/2. \end{aligned} \quad (9)$$

- (2) For a droplet in *shear flow* with shear rate Γ , $\text{Pe} \sim R^2$, and for a *rigid* interface, the coefficients B and β are

$$\begin{aligned} B &= B_3 \Gamma^{1/3} D^{2/3}, \\ \beta &= 1/3. \end{aligned} \quad (10)$$

For a droplet in a shear flow with a *mobile* interface, the coefficients B and β are

$$\begin{aligned} B &= B_4 \Gamma^{1/2} D^{1/2}, \\ \beta &= 0. \end{aligned} \quad (11)$$

In the above equations, B_1, B_2, B_3 , and B_4 are $O(1)$ constants. The conservation equation (7) can be solved by defining the nondimensional quantities $x(t) = (R_c(t)/R_c(0))$, $u = (R/R_c(t))$, and $\tau = (\beta + 2)\log x$, where $R_c(t) = (\alpha/\Delta(t))$ is the critical radius which separates growing and shrinking droplets. The rate of change of droplet radius is now given by

$$\frac{du^{\beta+2}}{d\tau} = \gamma(u - 1) - u^{\beta+2}, \quad (12)$$

where the parameter positive γ is

$$\gamma = \frac{B\alpha}{R_c(0)^{\beta+2}} \left(x^{\beta+1} \frac{dx}{dt} \right)^{-1}. \quad (13)$$

The above equation has a solution that is consistent with mass conservation only if γ and u approach finite values in the limit $\tau \rightarrow \infty$

$$\begin{aligned} u &\rightarrow \frac{\beta+2}{\beta+1}, \\ \gamma &\rightarrow \frac{(\beta+2)^{\beta+2}}{(\beta+1)^{\beta+1}}. \end{aligned} \quad (14)$$

The above results are in agreement with the results of the Lifshitz–Slyozov theory for the case $\beta = 1$. From the above results, the late stage growth law for the droplet radius can be obtained

$$x(t)^{\beta+2} = \left(\frac{\beta+1}{\beta+2} \right)^{\beta+1} \frac{B\alpha}{R_c(0)^{\beta+2}} t. \quad (15)$$

With this, the following scaling laws are obtained

(1) Sedimenting droplet, rigid interface
 $R_c(t) \propto t^{1/2}$. (16)

(2) Sedimenting droplet, mobile interface
 $R_c(t) \propto t^{2/3}$. (17)

(3) Sheared droplet, rigid interface
 $R_c(t) \propto t^{3/7}$. (18)

(4) Sheared droplet, mobile interface
 $R_c(t) \propto t^{1/2}$. (19)

The method used for determining the size distribution of the droplets is very similar to that used in the Lifshitz–Slyozov theory,¹⁵ and the important steps are briefly explained here. The first correction to the growth rate is determined using a Taylor series expansion in the parameter τ . The coefficient γ is expanded about its leading order approximation $\gamma_0 = (\beta + 2)^{(\beta + 2)} / (\beta + 1)^{(\beta + 1)}$ (14)

$$\gamma = \gamma_0 (1 - \epsilon(\tau)^2). \tag{20}$$

The conservation equation (12) correct to $O(\epsilon^2)$ is

$$\frac{du}{d\tau} = -\frac{\epsilon^2}{\beta + 1} - \frac{(\beta + 1)^2}{2(\beta + 2)} (u - u_0)^2, \tag{21}$$

where $u_0 = (\beta + 2) / (\beta + 1)$. The above equation is recast using the variables $z = (u - u_0) / \epsilon$ and $\eta = d\epsilon^{-1} / d\tau$ to give

$$\frac{2(\beta + 2)}{(\beta + 1)^2} \frac{1}{\epsilon} \frac{dz}{d\tau} = -z^2 - \frac{2(\beta + 2)}{(\beta + 1)^3} + \frac{2(\beta + 2)}{(\beta + 1)^2} z\eta. \tag{22}$$

Using conditions similar to those used by Lifshitz and Slyozov,¹⁵ it can be inferred that the solutions of the above equation are consistent with the conservation requirements only if z and η tend to finite values in the limit $\tau \rightarrow \infty$

$$\eta = \left[\frac{2(\beta + 1)}{\beta + 2} \right]^{1/2}, \quad z = \left[\frac{2(\beta + 2)}{(\beta + 1)^3} \right]^{1/2}. \tag{23}$$

This gives the first correction to the parameter γ

$$\gamma = \left[\frac{(\beta + 2)^{\beta + 2}}{(\beta + 1)^{\beta + 1}} \right] \left[1 - \frac{\beta + 2}{2(\beta + 1)\tau^2} \right]. \tag{24}$$

The probability distribution function for the droplet radii can be determined in a manner similar to that in the Lifshitz–Slyozov theory. The probability distribution function $P(u, \tau)$ is defined such that $P(u, \tau) du$ provides the probability of finding a droplet with reduced radius in the interval du about u at time τ . The conservation equation for the probability distribution is

$$\frac{\partial P}{\partial \tau} + \frac{\partial(vP)}{\partial u} = 0, \tag{25}$$

where

$$v = \frac{du}{d\tau} = \frac{u_0^{(\beta + 1)}}{u^{(\beta + 1)}} (u - 1) - \frac{u}{\beta + 2}. \tag{26}$$

The characteristic solution for the conservation equation (25) is

$$P(u, \tau) = \chi(\tau - f(u)) / (-v), \tag{27}$$

where

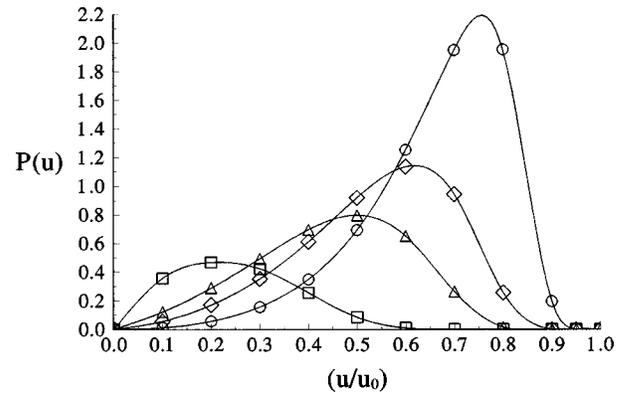


FIG. 1. $P_u(u)$ as a function of (u/u_0) for different values of the parameter β . (○)— $\beta=1$; (△)— $\beta=0$; (□)— $\beta=-1/2$; (◇)— $\beta=1/3$.

$$f(u) = \int_0^u du' [v(u')]^{-1} \tag{28}$$

and χ is a function to be determined.

Using a procedure similar to Lifshitz and Slyozov,⁸ it can be shown that in the solution for the function $P(u, \tau)$ in the late stages is

$$P(u, \tau) = A \exp\left(-\frac{3(\tau - f(u))}{\beta + 2}\right) / (-v(u)) \\ = A \exp\left(-\frac{3\tau}{\beta + 2}\right) P_u(u), \tag{29}$$

where A is the normalization constant.

Unlike in the Lifshitz–Slyozov theory for solid alloys, it is not possible to get an analytical solution for the probability distribution function $P_u(u)$ for general values of β . An analytical solution is possible for the case $\beta=0$

$$P_u(u) = \frac{16u}{(2-u)^5} \exp\left(\frac{-3u}{2-u}\right). \tag{30}$$

For other values of β , the solution can be obtained numerically, and the solution is finite only for $u < u_0$. The probability distribution function $P_u(u)$ is shown as a function of u for the three different cases, along with that for the Lifshitz–Slyozov theory ($\beta=1$), in Fig. 1.

The mean radius and the mean square of the drop radius can be determined

$$\langle R \rangle = \frac{R_c \int_0^{u_0} du u P_u(u)}{\int_0^{u_0} du P_u(u)}, \tag{31}$$

$$\langle R^2 \rangle = \frac{R_c^2 \int_0^{u_0} du u^2 P_u(u)}{\int_0^{u_0} du P_u(u)}. \tag{32}$$

The results for these parameters are summarized in Table I. It is seen, both from Table I and Fig. 1, that the drop size distribution is peaked at a lower value of (u/u_0) as the exponent β is decreased, and the ratio $(\langle R \rangle / R_c)$ also decreases. In addition, the polydispersity and the standard deviation in the drop size distribution are greater as the parameter β is decreased. The number of droplets per unit volume N can be determined from

TABLE I. Characteristics of the critical radius and the droplet size distribution function for different values of β . Case (a) corresponds to the Lifshitz–Slyozov theory for binary alloys, case (b) corresponds to a sedimenting droplet with rigid interface and a droplet in shear flow with a mobile interface, case (c) corresponds to a sedimenting droplet with a mobile interface and case (d) corresponds to a droplet in shear flow with a rigid interface. β is the exponent in Eq. (7), $R_c(t)$ is the critical radius, u_0 is the maximum value of $u = (R/R_c(t))$, γ_0 is the leading order value of the parameter γ in Eq. (13), and $\langle R \rangle$ and $\langle R^2 \rangle$ are the mean and mean square radii defined in Eqs. (31) and (32) and N is the number of droplets per unit volume defined in Eq. (33).

Case	(a)	(b)	(c)	(d)
$\beta =$	1	0	-1/2	1/3
$R_c(t) \propto$	$t^{(1/3)}$	$t^{(1/2)}$	$t^{(2/3)}$	$t^{(3/7)}$
$u_0 =$	(3/2)	2	3	(7/4)
$\gamma_0 =$	(27/4)	4	(3 $\sqrt{3}$ /2)	(7 $^{(7/3)}$ /3(4 $^{(4/3)}$))
$(\langle R \rangle / R_c) =$	1	(8/9)	0.7341	0.9428
$(\langle R^2 \rangle - \langle R \rangle^2) / (\langle R \rangle^2) =$	0.046 25	0.6875	1.5280	0.3953
$N \propto$	t^{-1}	$t^{-3/2}$	t^{-2}	$t^{-9/7}$

$$N = \int_0^{u_0} du P(u)$$

$$= A \exp\left(-\frac{3\tau}{\beta+2}\right)$$

$$\propto t^{-3/(\beta+2)}. \quad (33)$$

The rate of decrease of the number of droplets is accelerated due to convective effects.

III. CONCLUSIONS

The rate of droplet growth in a binary fluid was derived in the high Peclet number limit where convective effects, due to the sedimentation or shear flow, are strong compared to diffusion in the matrix. It was shown that this approximation is valid for droplets of size $\geq 10 \mu\text{m}$ for simple fluids, and for smaller droplets for complex fluids with a low diffusion coefficient and high viscosity. First, the concentration and velocity field for the droplet were analyzed, and scaling relations were derived for the flux of the minority phase into the droplet as a function of the matrix concentration and the characteristic velocity. These scaling relations were then inserted into the equation for the rate of change of the droplet radius, and relations for the rate of increase of the average radius of the droplet were derived.

The results indicate that the scaling laws for the droplet growth in the limit of high convection could be very different from that in the absence of convection. The four main scaling relations for sedimenting droplets and droplets in shear flow with different interfacial conditions are summarized in Eqs. (16)–(19). The results for a droplet with a mobile interface would apply for spinodal decomposition in pure fluids where the tangential velocity and stress are continuous across the interface. The results for a rigid interface

would apply to decomposition in fluids with surface active agents at the interface. In an interface with surfactants, tangential compression of the interface is resisted because it results in an increase in density of the surfactants from their equilibrium value. If the density of surfactant is sufficiently high that the surface is closely packed with surfactant molecules, further compression of the interface is not permitted and a no-slip boundary condition is appropriate.

It is found that the rate of growth is $O(\text{Pe}^{1/3})$ larger than that in the absence of convection for a droplet with a rigid interface, and $O(\text{Pe}^{1/2})$ larger than that in the absence of convection for a droplet with a mobile interface. The probability distribution function for the droplet size is also significantly affected by convective effects. The peak of the probability distribution function occurs at lower values of the scaled radius, and the polydispersity in the droplet size distribution is also higher in the presence of strong convective effects.

The enhancement of droplet growth due to fluid flow has been reported previously in literature. Baumberger, Perrot, and Beysens¹¹ observed that shear could enhance the nucleation and growth of droplets in a binary mixture, but their experiments were carried out in the early stages of growth, and the scaling law observed was $\langle R \rangle \propto t^{(0.6 \pm 0.1)}$. Though this is slightly higher than the predictions $\langle R \rangle \propto t^{(3/7)}$ and $\langle R \rangle \propto t^{(1/2)}$ of the present analysis, this does confirm that there is a significant increase in the scaling exponent due to the application of shear. In addition, they also observed an enhancement of the flux at the interface by a factor proportional to $\text{Pe}^{1/3}$, also in agreement with the predictions of this analysis for a rigid interface. However, it has also been speculated¹² that sedimentation could alter the scaling law for late stage growth from $\langle R \rangle \propto t^{(1/3)}$ in the absence of gravity to $\langle R \rangle \propto t$ in the presence of gravity. The present analysis indicates the scaling exponent could change at most from $\langle R \rangle \propto t^{(1/3)}$ to $\langle R \rangle \propto t^{(2/3)}$, though the rate of growth of droplets could be significantly increased.

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