Experimental study of the instability of the viscous flow past a flexible surface

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(Received 27 April 2000; accepted 19 October 2001)

The viscous instability in the flow past a soft material is experimentally studied. The experiment is carried out using the parallel plate geometry of a rheometer, and a sheet of polyacrylamide gel of thickness about 4.5 mm is placed on the bottom plate. The fluid, silicone oil, is placed on the surface of the gel and the top plate is lowered till a preset gap of thickness between 300 and 1000 μm is attained. The rheometer is operated in the stress controlled mode, where the stress is increased at a constant rate, and the strain rate and apparent viscosity (assuming the flow in the gap is laminar) are recorded. Care is taken to ensure the Reynolds number is less than 1, so that inertial effects are negligible. The experimental results show that there is an anomalous increase in the apparent viscosity, determined assuming the flow is laminar, at a certain strain rate. This indicates that the flow becomes unstable and undergoes a transition from a laminar flow to a more complicated flow profile. This transition is repeatable if the experiment is stopped before there is irreversible damage to the gel surface. The experimental results are compared with theoretical predictions, and quantitative agreement is found with no adjustable parameters for a range of gap thicknesses and gel moduli. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1427923]

I. INTRODUCTION

There has been a lot of work on the stability of fluid flows past compliant surfaces, mostly motivated by aerospace and marine applications. In these applications, it is desirable to delay transition from laminar to turbulent flow in order to reduce drag, and the possibility of using compliant coatings to reduce drag has been the motivation for these studies. The first experimental studies were carried out by Kramer,[1,2] who examined the possibility that the compliance of dolphins' skins could result in reduced drag. Theoretical studies were carried out by Benjamin[3,4] and Landahl,[5] who used a linear stability analysis to examine whether transition could be delayed. Benjamin represented the surface dynamics by a complex compliance (the ratio of the normal stress and normal displacement at the wall), while Landahl used the complex admittance in order to characterize the wall. They identified three classes of instabilities. In addition to the Tollmien–Schlichting modes in the flow past rigid surfaces, there are free surface waves (Class B waves) modified by the flow, and a class of modes analogous to the Kelvin–Helmholtz modes (Class C waves). The authors concluded that compliant coatings could reduce drag by increasing the critical Reynolds number and decreasing the growth rate, but reduction was unlikely for the parameters used in the experiments of Kramer.

More recently, Carpenter and Garrad[6,7] did linear stability studies on the flow past Kramer type coatings, and classified the instabilities into Tollmien–Schlichting instability and flow induced surface instability. They observed that the Tollmien–Schlichting instability and the flow induced surface instability could interact to give a powerful new instability. In addition, there is the static divergence, which is an absolute instability that precedes flutter in aerodynamic structures. The static divergence has been the subject of a lot of experimental research.[8,9] The compliant wall used in these studies is a soft coating such as PVC plasticol, and it is observed that waves appear at the surface and there is a significant increase in the drag force when the velocity is increased beyond a critical value. The onset speed depends on the thickness of the coating and its modulus of rigidity, indicating a coupling between the flow dynamics and the dynamics of the surface. There is less evidence for the flow induced surface instabilities.[10]

Biological systems form another important example where there is fluid flow past soft materials. However, in contrast to marine and aerospace applications, the Reynolds number is very low (typically less than 1), and the elasticity of membranes and tissues is about three orders of magnitude lower than that in engineering materials. The Reynolds number for the blood flow through arteries and veins varies between a minimum of 0.01 in the micro-capillaries and a maximum of 600 in the aorta.[11] The shear moduli of the walls of arteries and veins have a shear modulus which varies in the range 10–100 kPa, though this value is sensitive to the strain imposed in the rest state.[12]

The theoretical studies of Kumaran *et al.*[13] and Kumaran[14] have shown that the flow past a flexible surface could become unstable even in the limit of zero Reynolds number. The flow becomes unstable when the dimensionless number $(V\mu/GR)$ exceeds a critical value, and the mechanism of instability is the transport of energy from the mean flow to the fluctuations due to the shear work done by the mean flow at the surface. Here V is the characteristic fluid velocity, μ is the fluid viscosity, G is the shear modulus of the surface, and R is the fluid thickness. In addition, there are

instabilities at high Reynolds number, such as the inviscid instability (Kumaran,[15] Shankar and Kumaran[16]) and the wall mode instability[17,18] which are not continuations of the rigid tube modes, but are qualitatively different. The wall model used in these studies, and in earlier studies on flow past compliant surfaces,[19–21] is a continuum viscoelastic material, and the dynamics of the wall was described using linear elasticity equations.

There have been very few experiments which have reported anomalously large drag forces at low Reynolds numbers in the flow past soft materials. Krindel and Silberberg[22] studied the flow in a tube whose walls were made of polyacrylamide gel. They observed that there is an anomalous increase in the drag force even at Reynolds numbers of about 700 where the flow in a rigid tube is stable. However, there have been no systematic studies on the viscous instability in the flow past soft materials.

The present experiments were motivated by the theoretical studies of Kumaran *et al.*[13] and Kumaran,[14] which predicted that there could be an instability in the flow past a soft material even in the absence of inertia. Care was taken to ensure that the Reynolds number is less than 1. Silicone oil, a highly viscous fluid with viscosity 1 Pa s was used, and the characteristic length in the experiment was maintained in the 300–1000 μm range to reduce the Reynolds number. In addition, all of the parameters in the linear viscoelastic wall model used in the theoretical studies were experimentally measured, and so there are no unknown parameters in the experiments. These studies provide experimental evidence for the presence of a viscous instability in the flow past a soft material even in the absence of inertia, and the results are in quantitative agreement with theoretical studies without any adjustable parameters.

II. EXPERIMENTAL PROCEDURE

The parallel plate geometry in the Rheolyst AR 1000N rheometer was used for the experiments. This consists of a stationary bottom plate, and a rotating top plate of radius 2 cm. For viscosity measurements, the angular velocity and the torque on the top plate are recorded, and the *shear stress and strain rate at the outer edge of the top plate are calculated assuming the flow is laminar*. The rheometer can also be used for oscillatory measurements of the storage and loss modulus over a frequency range 10^{-4}–100 Hz. In this case, the stress and strain at the outer edge are determined from the torque and angular velocity measurements assuming the deformation is affine. The rheometer is usually operated in stress controlled mode, where the torque of the rotor is fixed, and the angular velocity is determined. However, it can also be operated in the strain rate controlled mode.

The fluid used in the experiments was commercial grade silicone oil manufactured by GE silicones of viscosity about 1 Pa s, which is about 1000 times the viscosity of water, and the fluid density was measured to be 0.996×10^3 kg/m^3. The viscosity of the oil was measured using the parallel plate geometry in the Rheolyst AR 1000N rheometer. First, the gap between the top and bottom plate is set to zero, and contact is detected by a normal force transducer in the bottom plate. The top plate is then raised and silicone oil is placed on the bottom plate. The top plate is lowered to a preset distance, usually between 300 and 700 μm from the bottom plate, and the oil in between the two plates is held in place by surface tension. The top plate is rotated with a constant torque, and the angular velocity is determined. From this, the stress and strain rate at the outer edge of the plate are determined assuming the flow is laminar, and the viscosity is calculated. The reported viscosity, shown as a function of shear rate for different gap thicknesses in Fig. 1, is nearly constant as the gap thickness increases above 300 μm.

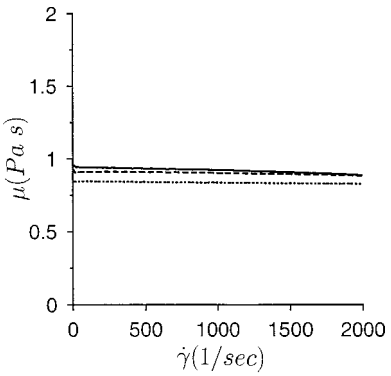

FIG. 1. Viscosity μ (Pa s) as a function of strain rate $\dot{\gamma}$ (1/s) for silicone oil for different gap thicknesses. The solid line shows the reading for a gap thickness of 700 μm, the long dash for gap thickness of 500 μm and the short dash for gap thickness of 300 μm.

The soft material used in the experiments was polyacrylamide gel, which was prepared as follows. The typical weight percentages of the constituents of the gelation mixture were N-acrylamide (monomer) 5%, Bis-acrylamide (cross linker) 0.05%, ammonium persulphate (initiator) 0.125%. The catalyst, TEMED, is added to the extent of 0.0625% by volume. In order to achieve these concentrations, stock solutions of 20% N-acrylamide and 0.5% bis-acrylamide by weight were prepared, and a solution of 0.5% by volume of TEMED was prepared. These were mixed in suitable quantities to get the desired concentrations, and the ammonium persulphate was weighed and then dissolved. In order to vary the viscoelastic properties, the concentrations of the catalyst and initiator were kept a constant throughout the experiments, while the concentrations of the monomer and crosslinker were changed in the range 5%–8% and 0.05%–0.08%. Two plane glass plates of side 5 cm were separated by spacers of thickness 4.5 mm on three sides, and clamped on these three sides, to form a rectangular container. The gelation mixture was poured into the container, and allowed to set for six hours. After the gelation was completed, the glass plates were removed and a rectangular sheet of gel of side ~5 cm$\times5$ cm and width ~4.5 mm was obtained. The gel density was measured as 1.013×10^3 kg/m^3.

The linear viscoelastic properties of this sheet were determined using a parallel plate geometry in the Rheolyst AR1000 rheometer. The gel was placed on the bottom plate of the rheometer, and the top plate was lowered until it made contact with the gel surface. Contact was detected when the normal force on the bottom plate increased to $0.2N$, which is

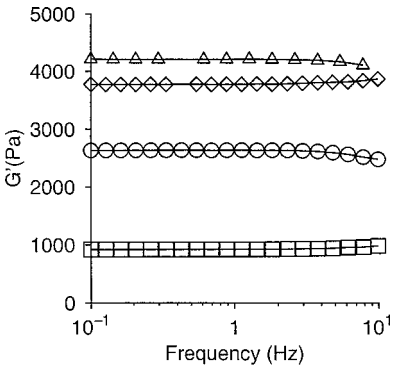

FIG. 2. Storage modulus G' (Pa) as a function of frequency ω (Hz) of four different gels. □—Monomer 5%, crosslinker 0.05%; ○—monomer 6%, crosslinker 0.06%; ◇—monomer 7%, crosslinker 0.07%; △—monomer 8%, crosslinker 0.08%.

the default instrument setting for detecting contact. The rheometer was then operated in the oscillatory mode, where an oscillatory stress was applied on the top plate and the angular displacement was measured. The storage and loss moduli were calculated assuming the deformation in the gap is affine. It was verified that the variations in the modulus were less than 2% when the top plate position was further lowered by 50 μm, indicating that contact was established between the two plates. The frequency spectrum of a typical gel cast using the above concentrations is shown in Fig. 2 in the range 0.1–10 Hz. It was not possible to obtain the moduli at higher frequencies, because there was slip between the top plate and the gel, and because the maximum operating frequency of the instrument is 100 Hz and the errors in measurement become significant as this frequency is approached. The storage modulus G' shows a distinct plateau region in the frequency range 0.1–10 Hz. The loss modulus G'' does not show a linear dependence on the frequency in this range, but it does increase with frequency. An important parameter in the theoretical studies is the gel viscosity $\mu_g=(G''/\omega)$. The frequency spectrum of this parameter, shown as a function of frequency in Fig. 3, is nearly independent of gel composition even though the storage modulus shows a variation from 1000–4000 Pa. In addition,

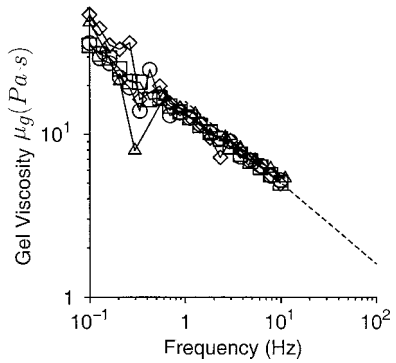

FIG. 3. Gel viscosity $\mu_g=(G''/\omega)$ (Pa s) as a function of frequency ω (Hz) for four different gels. The solid line shows the relation $\mu_g=16\omega^{-1/2}$, which was used for comparison with theoretical results. □—Monomer 5%, crosslinker 0.05%; ○—monomer 6%, crosslinker 0.06%; ◇—monomer 7%, crosslinker 0.07%; △—monomer 8%, crosslinker 0.08%.

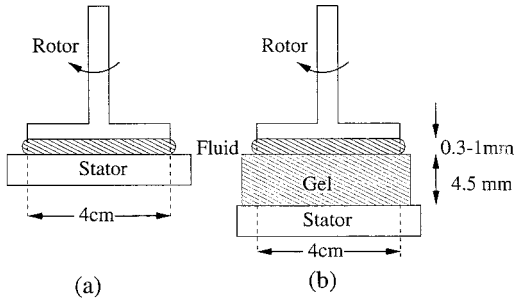

FIG. 4. Rheometer used in the experiments: (a) unmodified; (b) modified.

it shows a power law scaling $\mu_g \propto \omega^{-1/2}$ in the frequency range 1–10 Hz. This is characteristic of many soft solids such as polymer gels emulsions and liquid crystalline materials at high frequencies (Liu *et al.*[23]). The theoretical predictions for the parameter values used in the experiments indicate that the frequency of the most unstable perturbations varies in the range 10–500 Hz, and so it is necessary to determine the relative viscosity in this range. Since experimental measurements could not be made beyond 10 Hz, and because of the observation that the gel viscosity is proportional to $\omega^{-1/2}$ at the higher end of frequencies probed in the experiments as expected for soft solids, the scaling law $\mu_g = 16\omega^{-1/2}$ (which corresponds to the broken line in Fig. 3) was used.

In order to study the flow past a flexible surface, the rheometer was modified as shown in Fig. 4. The polymer gel was placed on the bottom plate of the rheometer. The thickness of the gel H was determined using the zero gap facility in the rheometer as follows. In order to accurately set the gap between the two plates, it is first necessary to find the top plate position at which the gap is zero. This is done by lowering the top plate until it touches the bottom plate, and the contact between the plates is sensed by a normal stress sensor on the bottom plate. The default value of the normal stress for detecting contact is $0.2N$. In the experiments on the flow past the gel, the contact between the top and bottom plates was first detected using the zero gap facility. The top plate was then raised and the polymer gel was placed on the bottom plate. The top plate was lowered until it made contact with the gel, and contact assumed when the normal stress reached the default value of $0.2N$. The height of the top plate at this stage provided the gel thickness, which varied between 4.2 and 4.7 mm for all the experiments reported here.

After determining the height, the top plate was raised and silicone oil was placed on the surface of the gel. The top plate was then lowered until it was at a preset distance, usually 1000–300 μm, from the gel surface. Care was taken to ensure that this gap was filled with fluid. The rheometer was then operated in the stress controlled mode, where the shear stress is increased as a linear function of time and the strain rate and apparent viscosity were recorded. The stress controlled mode was chosen in preference to the more natural strain rate controlled mode because the control in the strain rate controlled mode is achieved by a feedback loop, which makes it difficult to control the strain rate when the transition occurs. The result of a typical experiment is shown in Fig. 5.

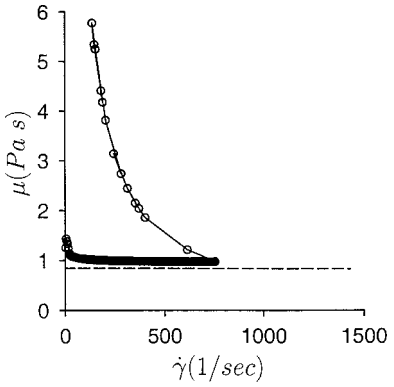

FIG. 5. Apparent viscosity μ (Pa s) as a function of strain rate $\dot{\gamma}$ (1/s) for the flow past a gel of shear modulus 2354 Pa and thickness 4490 μm. The fluid thickness is 300 μm, and the shear stress was increased at the rate of 4 Pa/s. The broken curve shows the result for the same fluid thickness between rigid surfaces.

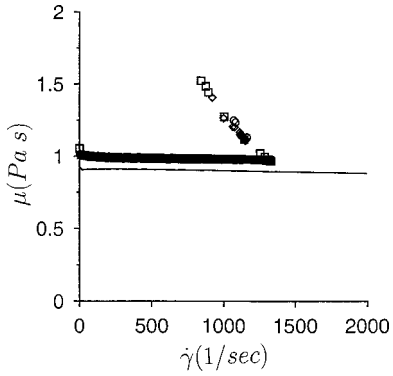

FIG. 6. The variation of the apparent viscosity μ with the strain rate in a controlled stress experiment. The solid curve shows the viscosity as a function of strain rate for the flow between two rigid surfaces. The symbols show the results for the flow past a gel of thickness 4600 μm and storage modulus 3113 Pa, and fluid thickness 500 μm. ○—Shear stress increased at 8 Pa/s; □—shear stress increased at 7 Pa/s up to 1250 Pa, and then at 0.5 Pa/s; ◇—shear stress increased at 7 Pa/s up to 1250 Pa, and then at 0.25 Pa/s.

In the case of flow between two rigid surfaces, though there is some initial variation in the viscosity, probably due to instrumental limitations at low strain rates, it is observed that the apparent viscosity is independent of the strain rate up to the maximum used in the experiment 1500 1/s. In the case of flow on the gel surface, however, it is observed that the viscosity is close to that between rigid surfaces at low strain rates, and the difference between the two is probably due to a variation in the gap thickness due to gel compression. However, there is a sharp transition and the apparent viscosity increases sharply at a strain rate of 700 1/s. This indicates that the flow in the gap is no longer has circular streamlines, as assumed while calculating the viscosity, but has a more complicated form. After transition, there is an increase in the apparent viscosity. This is accompanied by a decrease in the apparent strain rate *calculated assuming that the flow is laminar and has parallel streamlines in the cross stream direction*, because the stress is maintained at a constant value in the experiments. However, it should be noted that the viscosity and the strain rate after the transition point are not indicative of their microscopic values in the fluid because the flow is no longer laminar, and the only parameter of significance is the strain rate and stress at which the transition takes place. It is important to note that in the rheometer, the quantities measured are the *torque and angular velocity* of the top plate, and from these the strain rate and shear stress at the outer edge are determined assuming the *flow is laminar with parallel streamlines in the cross stream direction*, and any change in the form of the velocity field to a more complicated form with surface oscillations would give an apparent viscosity that is higher than the intrinsic viscosity.

If the experiment is allowed to progress, the surface of the gel gets damaged and small pieces break off from the surface. However, the transition point can be obtained repeatably if the experiment is stopped before damage is caused to the gel. Figure 6 shows the results for three different runs on the same gel surface, where the experiment was stopped before the gel was damaged. This implies that the transition itself is repeatable, and not the result of some irreversible change in the fluid or gel properties.

Inertial effects are not important in these experiments, because the Reynolds number is at most 0.1. Therefore, it is appropriate to compare the present results with the linear stability analysis of the Couette flow past a flexible surface.[13] Though the present geometry is not a two dimensional flow, the gap thickness (300–1000 μm) is small compared to the radius of the top plate (2 cm). Consequently, it is expected that the laminar flow is nearly two dimensional near the edge of the plate. Further, the onset of an instability should occur at the edge of the plate where the velocity is a maximum.

Since the shear stress was increased linearly during the experiments, it is important to verify that the time scale for the increase of the shear stress is long compared to the inverse of the shear rate of the flow, or the time scale for the gel dynamics, so that the results obtained here correspond to the results that would be obtained in a steady experiment. This verification was carried out in two ways.

(1) It was first verified experimentally that the results for the transition are independent of the rate of change of the shear stress by carrying out different experiments on the same gel with different rates of increase of the shear stress. An example of this is shown in Fig. 6. Here, the three symbols represent the transition values of the strain rates obtained for different rates of increase of shear stress varying from 0.25 to 8 Pa/s. It is observed that there is no variation in the transition strain rate when rate of change of shear stress is changed by a factor of 32.
(2) The time scale for the variation of the shear stress was compared with the other relevant time scales, which are the inverse of the strain rate and the ratio (μ_g/G') for the gel dynamics. The minimum strain rate at which the transition was observed in the present experiments was $\sim 200\ s^{-1}$. The flow time scale, which is the inverse of the strain rate, is at most 5×10^{-3} s. Assuming that the stress is increased linearly at the rate of 8 Pa/s (which was the maximum value used in the experiments), the increase in the stress during a time period of 5×10^{-3} s is 0.04 Pa, which is two orders of magnitude lower than

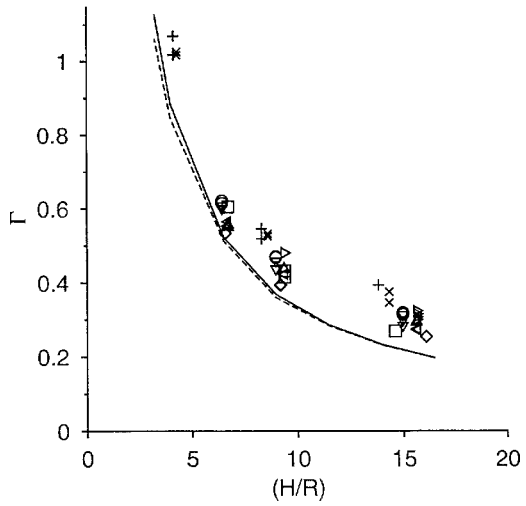

FIG. 7. The ratio (τ/G') of the transition value of the shear stress τ and the shear modulus G' as a function of the ratio of gel and fluid thicknesses (H/R) for different gels. The transition stress is the stress at the turning points in Figs. 6 and 7. The solid and broken lines represent the theoretically predicted transition stress for $G'=1000$ Pa and $G'=4000$ Pa for different values of (H/R) from Ref. 13. ○—$H=4490\ \mu$m, $G'=2305$ Pa; □—$H=4699\ \mu$m, $G'=3788$ Pa; ◇—$H=4600\ \mu$m, $G'=4214$ Pa; △—$H=4690\ \mu$m, $G'=2642$ Pa; ▽—$H=4490\ \mu$m, $G'=2354$ Pa; ◁—$H=4678\ \mu$m, $G'=4040$ Pa; ▷—$H=4690\ \mu$m, $G'=3595$ Pa; +—$H=4150\ \mu$m, $G'=947$ Pa; ×—$H=4300\ \mu$m, $G'=1027$ Pa.

the minimum stress of 200 Pa at which the transition was observed. The time scale for the gel dynamics, (μ_g/G'), is 1.13×10^{-3} s for the softest gels used here ($G'\sim1000$ Pa). This is even smaller than the flow time scale.

This indicates that the increase in the shear stress is negligible over time scales comparable to the flow time or the time scale of the gel dynamics, and the results could be compared with those for a steady flow past a flexible surface.

The linear stability analysis predicts that the flow becomes unstable when the dimensionless parameter $\Gamma=(V\mu/G'R)$ increases beyond a critical value for a given set of ratio of thickness of the gel and fluid (H/R) and viscosity ratio (μ_g/μ). In the present case, the parameter $\Gamma=(\tau/G')$, where τ is the shear stress at the outer edge of the plate in the laminar flow as recorded by the rheometer. Figure 7 shows the ratio (τ/G') as a function of (H/R) for many different gels with shear moduli in the range 900–4200 Pa, where τ is the shear stress at the turning points in Figs. 6 and 7. It is observed that the parameter (τ/G') varies only by about 5%–7% even though shear modulus changes by a factor of 4. The variation in the experimentally measured values of (τ/G') could be due to variations in the gel thickness. Since the gel is a soft material, it is not possible to determine the thickness using measuring instruments traditionally used for hard materials. In the present case, the thickness was measured using the normal force measurement facility in the rheometer, using a preset value of the normal force for measuring contact. There could be a variation in the gel thickness due to compression during flow. For the (H/R) ratios used here, a variation of 1% in the gel thickness would result in an error of more than 10% in the gap thickness for a gap thickness of 300 μm, and so this could affect the calculations. Considering the above uncertainties in the experimental measurements, the variation in the results is smaller than that which would be expected from an assessment of the experimental errors.

Figure 7 also shows the theoretical predictions[13] for the variation of the transition stress with (H/R) for shear modulus $G'=1000$ Pa and $G'=4000$ Pa, which is the range of moduli used in the present experiments. These results were determined using the following iterative procedure. For a given (H/R), a value of viscosity ratio (μ_g/μ) was assumed, and the critical parameter Γ_c, the wave number of the most unstable mode k_c and the frequency of the most unstable mode ω_c were calculated. The viscosity of the gel μ_g was determined at the frequency ω_c using the power law relation in Fig. 3. The viscosity ratio was determined using the new relative viscosity, and a new critical value Γ_c was determined for this viscosity ratio. This procedure was continued till convergence was obtained for the value of the viscosity ratio. The critical value of Γ does shows very little dependence on the shear modulus G' for the following reason. The theoretical studies predict that at a given gel viscosity, Γ_c is constant with an increase in the modulus. However, in this case, as the shear modulus is increased, the characteristic frequency of gel oscillations increases. This results in a lower gel viscosity due to the $\omega^{-1/2}$ scaling of gel viscosity with frequency. Since Γ_c decreases slowly with a decrease in the gel viscosity, an increase in the shear modulus G' causes a small decrease in Γ_c.

The good agreement between experimental results and theoretical predictions confirms that the transition in the type of flow is due to a viscous instability in the flow past a flexible surface.

III. CONCLUSIONS

The present study has probed the possibility of an instability in the Couette flow past a flexible surface in the limit of zero Reynolds number. The experimental results were compared with the theoretical predictions of Kumaran *et al.*,[13] and all the model parameters used in the viscoelastic wall model were independently measured in the experiments. The experimental observations confirm the existence of an instability in the flow past a soft material even in the absence of inertial effects, and the results are in quantitative agreement with the theoretical predictions. These results could have significant implications for flows involving biological systems, since most of these are at low Reynolds numbers where inertial effects are not important, and involve soft materials. In particular, the results indicate that the effect of flow induced stresses at a soft surface could be different from stresses induced by other mechanisms due to the coupling between the flow dynamics and the dynamics of the soft material.

Another important result of the experiments, which was not reported in the experimental studies, is the absence of a nonlaminar steady state after transition. Even though efforts were made in the experiments to maintain the shear stress at

a value slightly higher than the laminar value, it was found that in all cases the apparent viscosity increases to a high enough value that eventually damage is caused to the surface. This seems to indicate that the transition is subcritical, and there is no nearby equilibrium state, unlike in the case of Rayleigh–Benard or Taylor–Couette instabilities. This is consistent with the weakly nonlinear analysis of Shankar and Kumaran,[6,24] where a Landau equation was derived for the Couette flow of a fluid past a gel. In that study, it was found that the Landau constant is always positive for the present case, indicating that the transition is subcritical. However, there are still unanswered questions regarding the nature of the transition as the Reynolds number is increased.

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